Konvergensordning

Newtons: $x_{k+1} = \underbrace{x_k - \frac{f(x_k)}{f'(x_k)}}_{g(x_k)}$

Sekant: $x_{k+1} = \underbrace{x_k - f(x_k)\frac{x_{k-1} - x_k}{f(x_{k-1}) - f(x_k)}}_{g(x_k, x_{k-1})}$

$$|x_{k+1} - \underbrace{x_*}_{\text{exakt}}| \le \underbrace{C}_{\substack{\text{konst} \\ < 1}} |x_k - x_*|$$

Konv. ordn.

$r = 1: |x_{k+1} - x_*| \le C |x_k - x_*|$

$r = 2: |x_{k+1} - x_*| \le C |x_k - x_*|^2$

När $\underbrace{r = 1}_{\text{linjär konvergens}}$: $k = 0: |x_1 - x_*| \le \underbrace{C}_{<1} |x_0 - x_*|$

$k = 1: |x_2 - x_*| \le \underbrace{C}_{<1} |x_1 - x_*| \le$

$\le \underbrace{C^2}_{<1} |x_0 - x_*|$

$\cdots$

$k = n: |x_n - x_*| \le \underbrace{C^n}_{<1} |x_0 - x_*|$

Konvergensordning

$\boxed{r=2:}$ $\quad |x_{k+1} - x_*| \leq C|x_k - x_*|^2$

kvadratisk konvergent

$k=0: \quad |x_1 - x_*| \leq C|x_0 - x_*|^2$

$k=1: \quad |x_2 - x_*| \leq C|x_1 - x_*|^2 \leq$

$$\leq C \cdot \left[\left(C|x_0 - x_*|^2\right)^2\right]$$

$$= C^3 \cdot |x_0 - x_*|^4$$

$k=2: \quad |x_3 - x_*| \leq C|x_2 - x_*|^2 \leq$

$$\leq C \cdot \left(\left(C^3 \cdot |x_0 - x_*|^4\right)^2\right)$$

$$= C^7 \cdot |x_0 - x_*|^8$$

$k=3: \quad |x_4 - x_*| \leq C \cdot |x_3 - x_*|^2 \leq$

$$\leq C \cdot \left(\left(C^7 \cdot |x_0 - x_*|^8\right)^2\right)$$

$$= C^{15} \cdot |x_0 - x_*|^{16}$$

$\cdots$

$$\boxed{n := k+1 \quad |x_n - x_*| \leq C^{2^n - 1} \cdot |x_0 - x_*|^{2^n}}$$

$$x^{k+1} = g(x^k)$$

Exakt $\leftarrow$ $$x^* = g(x^*)$$

$$\boxed{x^* - g(x^*) = 0}$$

$$\boxed{g(x) = x^2}$$

$$\boxed{x^{k+1} = g(x^k) = (x^k)^2}$$

$$g(x^*) = (x^*)^2$$

$$x^* - (x^*)^2 = 0$$

$$\underbrace{x^*}_{=0}(1 - \underbrace{x^*}_{=1}) = 0$$

$x^* = 0$, $x^* = 1$, $|g'(x^*)| < 1$

$$|g'(x_*)| < 1 -$$

konvergens

$$g(x) = x^2$$

$$g'(x) = 2x$$

$$|g'(x^*)| = |g'(0)| = 0 < 1$$

– konvergens

$$|g'(x^*)| = |g'(1)| = 2 > 1$$

divergens

$$f(x) = x^2 = 0$$

$$f(x) = x^2 - x + x \rightarrow$$

$$\rightarrow x = x^2 + x \; g(x)$$

Sätt upp Newtons metod

$$f_1(x,y) = (\cos x)^2 \sin y - \sin x^2 = 0$$

$$f_2(x,y) = (\sin y)^2 \cos x - \cos y^2 = 0$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 2\cos x \cdot (-\sin x) \cdot \sin y - 2x \cdot \cos x^2; \\ (\sin y)^2 \cdot (-\sin x); \end{bmatrix}$$

$$(\cos x)^2 \cos y$$

$$2 \sin y \cdot \cos y \cdot \cos x$$

$$- 2y \cdot (-\sin y^2)$$

$$J_{(x^k, y^k)} = \begin{bmatrix} 2\cos x^k \cdot (-\sin x^k) \cdot \sin y^k - 2x^k \cdot \cos (x^k)^2; & (\cos x^k)^2 \cos y^k \\ -(\sin y^k)^2 (\sin x^k) & ; 2 \sin y^k \cdot \cos y^k \cdot \cos x^k + 2y^k \cdot \sin (y^k)^2 \end{bmatrix}$$

Newtons:

$$\begin{bmatrix} x^{k+1} \\ y^{k+1} \end{bmatrix} = \begin{bmatrix} x^k \\ y^k \end{bmatrix} - J^{-1}(x^k, y^k) \cdot \begin{bmatrix} f_1(x^k, y^k) \\ f_2(x^k, y^k) \end{bmatrix}$$