Minstakvadratproblem: normalekvationerna

$$\min_{x} \|Ax - b\|_2^2$$

Normalekvationerna: $\underbrace{A^TA}_{\text{ny matris}}^{n}\, x = \underbrace{A^Tb}_{\text{ny vektor}}^{n}$

$$n\left\{\overbrace{\begin{bmatrix} & A^T & \end{bmatrix}}^{m}\right. \cdot\; m\left\{\overbrace{\begin{bmatrix} & A & \end{bmatrix}}^{n}\right. = n\left\{\overbrace{\begin{bmatrix} & A^TA & \end{bmatrix}}^{n}\right.$$

$$A^Tb = n\left\{\overbrace{\begin{bmatrix} & A^T & \end{bmatrix}}^{m}\right. \cdot \left.\begin{bmatrix} b \end{bmatrix}\right\}m = \left.\begin{bmatrix} A^Tb \end{bmatrix}\right\}n$$

$$n\left\{\overbrace{\begin{bmatrix} & A^TA & \end{bmatrix}}^{n}\right. \cdot \left.\begin{bmatrix} x \end{bmatrix}\right\}n = \left.\begin{bmatrix} A^Tb \end{bmatrix}\right\}n$$

$$x = \underbrace{(A^TA)^{-1}A^T}_{A^+}b = A^+b$$

Konditionstal för minstakvadratproblem

Vi har 2 problem:

1) $\min\limits_{x} \|Ax - b\|_2^2$ 2) $\min\limits_{y} \|Ay - (b+f)\|_2^2$

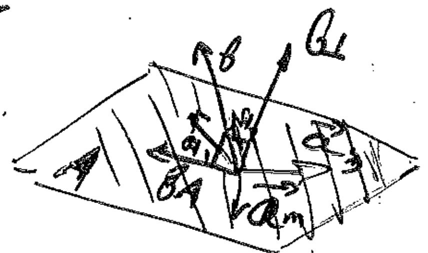
Lösning:

1) $x = \underbrace{(A^TA)^{-1}A^T}_{A^+} b = A^+ b$ 2) $y = \underbrace{(A^TA)^{-1}A^T}_{A^+}(b+f) = A^+(b+f)$

A^+ är pseudoinversen.

$A^+ = \mathrm{pinv}(A)$

$y - x = A^+(b+f) - A^+b = A^+f$



(*) $\|y-x\|_2 \leq \|A^+\|_2 \cdot \|f\|_2$

Använder: $Ax = b_A$ $\qquad \vec{b} = b_A + b_\perp$

$r = Ax - b = Ax - (b_A + b_\perp) = -b_\perp$

(**) $\|b_A\| \leq \|Ax\|_2 \leq \|A\|_2 \cdot \|x\|_2 \Rightarrow \dfrac{1}{\|x\|_2} \leq \dfrac{\|A\|_2}{\|b_A\|_2}$

Från (*) och (**):

$$\boxed{\frac{\|y-x\|_2}{\|x\|_2} \leq \underbrace{\|A\|_2 \cdot \|A^+\|_2}_{K_2(A)} \cdot \frac{\|f\|_2}{\|b_A\|_2}}$$

$K_2(A)$ → konditionstalet för minstakvadratproblem

Kan skrivas som:

$$\boxed{\frac{\|y-x\|_2}{\|x\|_2} \leq K_2(A)\frac{\|b\|_2}{\|b_A\|_2} \cdot \frac{\|f\|_2}{\|b\|_2}}$$

Practical error bounds

$$\min_x \|Ax - b\|_2^2$$

Lösning: $x = \underbrace{(A^TA)^{-1}A^T}_{A^+}b = A^+b$

$A\tilde{x} - b = r \rightarrow$ residual

$A\tilde{x} = r + b$

$A^TA\tilde{x} = A^T(r+b)$

$\tilde{x} = \underbrace{(A^TA)^{-1}A^T}_{A^+}(r+b) = A^+(r+b)$

$x - \tilde{x} = A^+b - A^+(r+b) = -A^+r$

$\|x - \tilde{x}\|_2 \leq \|A^+\|_2 \cdot \|r\|_2$ Mult. and divide by $\frac{\|A\|_2}{\|A\|_2}$:

relative error $\frac{\|x - \tilde{x}\|_2}{\|\tilde{x}\|_2} \leq \underbrace{\|A\|_2 \cdot \|A^+\|_2}_{K_2(A)} \cdot \frac{\|r\|_2}{\|b\|_2} = K_2(A) \frac{\|r\|_2}{\|b\|_2}$

$$\boxed{\frac{\|x - \tilde{x}\|_2}{\|x\|_2} \leq K_2(A) \frac{\|b\|_2}{\|bA\|_2} \cdot \frac{\|r\|_2}{\|b\|_2}}$$

Modellen: $b = e^{A - B/T + C/T^2}$

!!! känner $\rightarrow$ $$\log b = A_{-?} - \frac{B^{-?}}{T} + \frac{C^{-?}}{T^2}$$

Mult. till T^2:

rhs, kan räknas
$$T^2 \log b = \overset{?}{A}\, T^2 - \overset{?}{B}\, T + \overset{?}{C} \cdot 1 =$$
$$= X_1 \cdot 1 - X_2 \cdot T + X_3 \cdot T^2$$

$x_1 := C; \; X_2 := B; \; X_3 := A$

$$m\left\{\begin{bmatrix} 1; & -T_1; & T_1^2 \\ \vdots & & \\ 1; & -T_m; & T_m^2 \end{bmatrix}\right. \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = m\left\{\begin{bmatrix} T_1^2 \log b_1 \\ \vdots \\ T_m^2 \log b_m \end{bmatrix}\right.$$

m är antalet punkter var vi kan mäta T.

$$\min_{x=(x_1,x_2,x_3)} \| A x - b \|_2^2, \quad \text{var}$$

$$A = \begin{bmatrix} 1; & -T_1; & T_1^2 \\ \vdots & & \\ 1; & -T_m, & T_m^2 \end{bmatrix}; \quad x = \begin{bmatrix} C \\ B \\ A \end{bmatrix}$$

$$b = \begin{bmatrix} T_1^2 \log b_1 \\ \vdots \\ T_m^2 \log b_m \end{bmatrix}$$

$$\log b = -A + B\log T + \frac{C}{T^2}$$

Mult. T^2:

$$T^2 \cdot \log b = -A \cdot T^2 + B \cdot T^2 \log T + C;$$

$$X_1 = A; \quad X_2 = B; \quad X_3 = C$$

$$T^2 \cdot \log b = -X_1 \cdot T^2 + X_2 \cdot T^2 \log T + X_3 \cdot 1$$

$$\begin{bmatrix} -T_1^2; & T_1^2 \log T_1; & 1 \\ \vdots & & \\ -T_m^2; & T_m^2 \log T_m; & 1 \end{bmatrix} \cdot X = \begin{bmatrix} T_1^2 \log b_1 \\ \vdots \\ T_m^2 \log b_m \end{bmatrix}; \quad \min_X \|Ax - b\|_2^2$$

$$T\log B = \underbrace{x_1}_{T_0}\,;\ \log B + \underbrace{x_2}_{\log A}\,;\ T + \underbrace{x_3}_{E - T_0\log A}\,;\ 1$$

$$\overbrace{\begin{bmatrix} \log B_1 & T_1 & 1 \\ \vdots & \vdots & \vdots \\ \log B_n & T_n & 1 \end{bmatrix}}^{A} \cdot \overbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}^{x} = \underbrace{\begin{bmatrix} T_1 \log B_1 \\ \vdots \\ T_n \log B_n \end{bmatrix}}_{b}$$

$$\left[\ \min_{x} \|A \cdot x - b\|_2^2\ \right]$$

Modelekv. är: $A^{?}\, e^{\frac{E^{?}}{T - T_0}} = B$ (vi vet; vi vet)

$$\log A + \frac{E}{T - T_0} = \log B;$$

$$\log A^{?}(T - T_0) + E^{?} = (T - T_0)\log B = T\log B^{\text{vi vet}} - T_0\log B^{?}$$

$$\log A\, T - \log A\, T_0 + E + T_0 \log B = T\log B$$