

# Gaussian elimination with partial pivoting

Find pivot element in the 1-st column

0.02	0.01	0	0	0.02
1	2	1	0	1
0	1	2	1	4
0	0	100	200	800

To zero out  $a_{21}$  compute  $m_{21} = \frac{a_{21}}{a_{11}} = \frac{0.02}{1} = 0.02$

$E_2 - m_{21}E_1$   
zero out

1	2	1	0	1
0.02	0.01	0	0	0.02
0	1	2	1	4
0	0	100	200	800

fix this line and perform 1 step of Gauss elimination

Perform now Gaussian elimination 1 step  
then:  $m_{21} = \frac{a_{21}}{a_{11}}$

Find pivot element

1	2	1	0	1
0	-0.03	-0.02	0	0
0	1	2	1	4
0	0	100	200	800

fixed line  
 $(E_2 - m_{21}E_1)$

$m_{32} = \frac{a_{32}}{a_{22}} = \frac{2}{1} = 2$   
 $m_{33} = \frac{a_{33}}{a_{23}} = \frac{-0.02}{2} = -0.01$   
 $m_{34} = \frac{a_{34}}{a_{24}} = \frac{0}{1} = 0$   
 $m_{35} = \frac{a_{35}}{a_{25}} = \frac{0}{4} = 0$

1	2	1	0	1
0	1	2	1	4
0	-0.03	-0.02	0	0
0	0	100	200	800

already fixed line  
fix now this line

2)  $E_3 - m_{32}E_2$  Perform 1 step of Gaussian elimination to zero out

$= [0; -0.03; -0.02; 0; 0]$   
 $= [0.03 \cdot 1; -0.03 \cdot 2; -0.03 \cdot 1; -0.03 \cdot 4]$   
 $= [0; 0; 0.04; 0.03 | 0.12]$

1	2	1	0	1	
0	1	2	1	4	
0	0	0.04	0.03	0.12	
0	0	100	200	800	

→ fixed  
 → fixed

find pivot element in the 3-rd column

⇓

1	2	1	0	1	
0	1	2	1	4	
0	0	100	200	800	
0	0	0.04	0.03	0.12	

→ fixed  
 → fixed

Perform 1 step of Gauss elim. to zero out 0.04: 1)  $m_{43} = \frac{0.04}{100} =$

⇓

1	2	1	0	1	
0	1	2	1	4	
0	0	100	200	800	
0	0	0	-0.05	-0.2	

[0; 0; 0.04; 0.12]  
 $\begin{bmatrix} 0.04 \\ 100 \cdot 100 + \frac{0.04}{100} \cdot 200 \end{bmatrix}$   
 $\frac{0.04}{100} \cdot 800$

Obtain solution by backward substitution:

$$\begin{aligned}
 -0.05 x_4 &= -0.2 \Rightarrow x_4 = 4 \\
 100 x_3 + 200 x_4 &= 800 \Rightarrow x_3 = 0 \\
 x_2 + 2 x_3 + x_4 &= 4 \Rightarrow x_2 = 0 \\
 x_1 + 2 x_2 + x_3 + 1 &= 1 \Rightarrow x_1 = 0
 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$