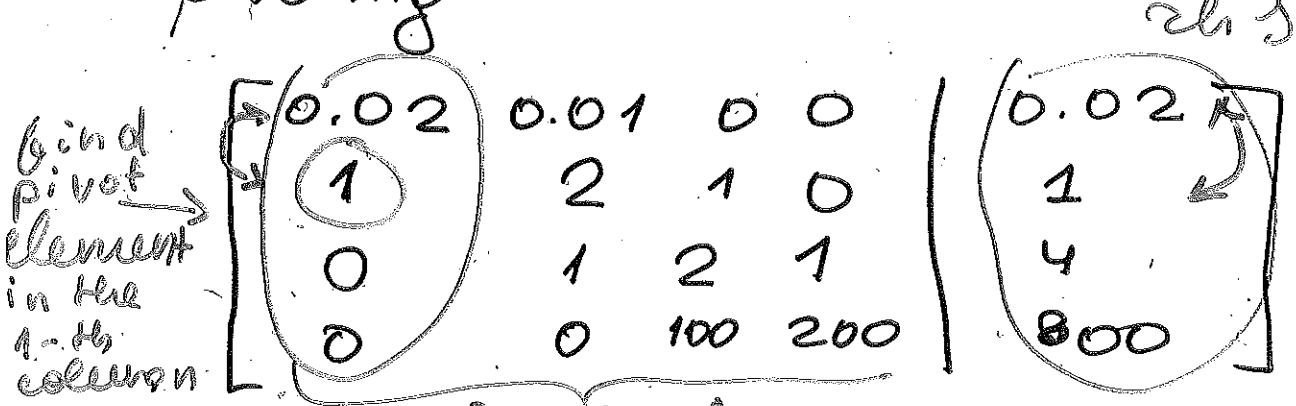
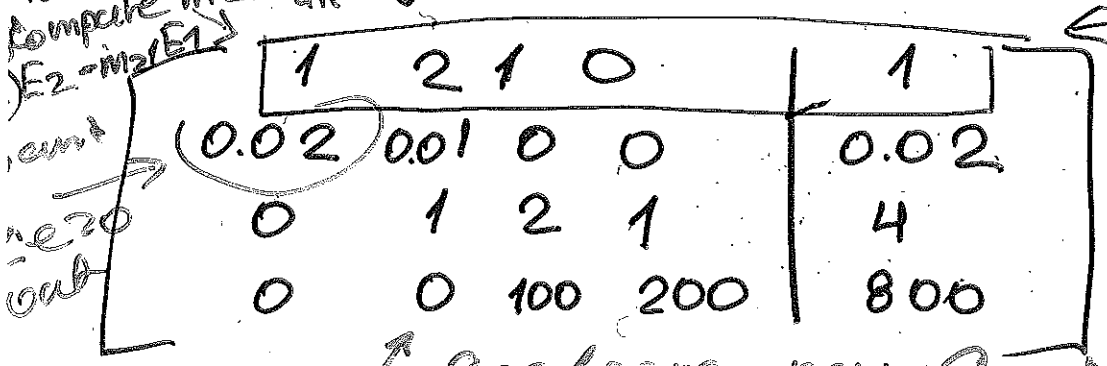
Gaussian elimination with partial pivoting

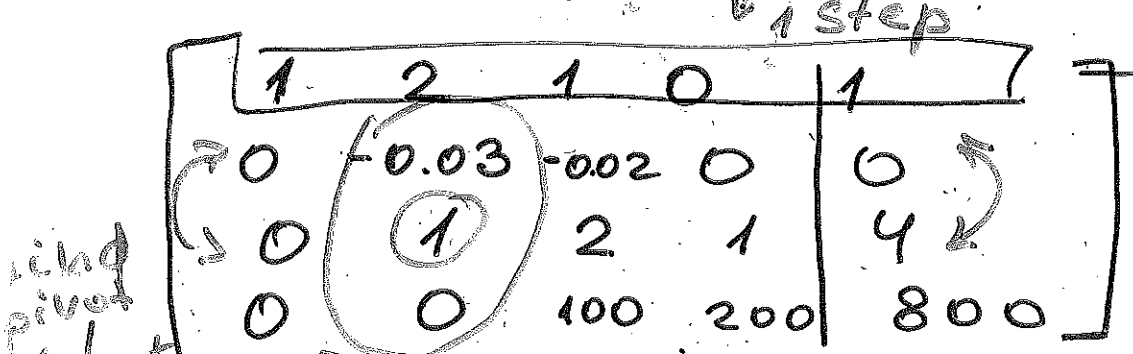


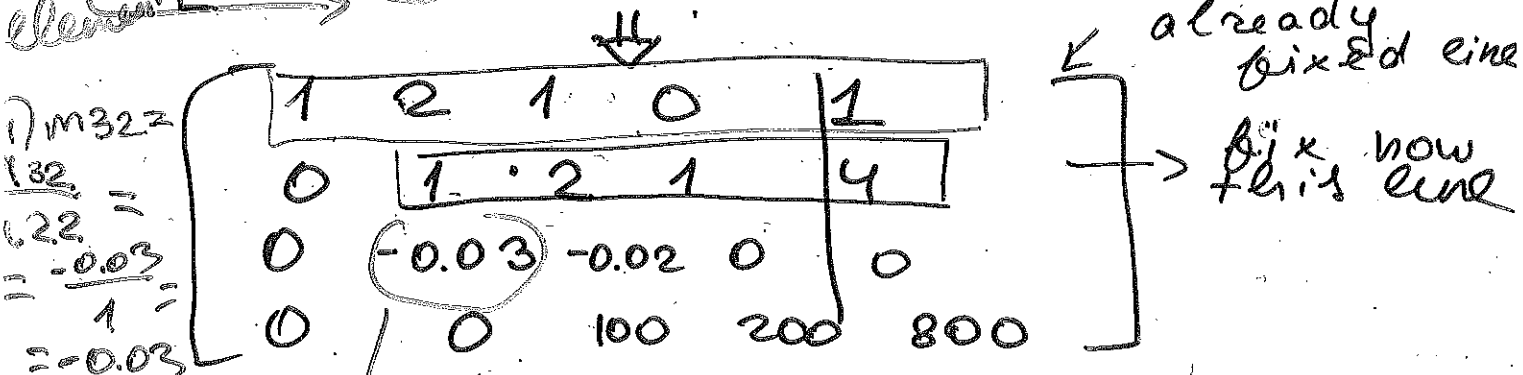
To zero out a_{21}: compute $m_{21} = \frac{a_{21}}{a_{11}} = \frac{0.02}{0.02} = 1$

Compute $(E_2 - m_{21}E_1)$



Perform 1 step of Gaussian elimination: $m_{21} = \frac{a_{21}}{a_{11}}$





2) $E_3 - m_{32}E_2$ — Perform 1 Step of Gaussian elimination to zero out -0.03

$= [0; -0.03; -0.02; 0 \mid 0] - [0; 0.03 \cdot 1; -0.03 \cdot 2; -0.03 \cdot 1 \mid -0.03 \cdot 4]$

$= [0; 0; 0.04; 0.03 \mid 0.12]$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 0.04 & 0.03 & 0.12 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

→ fixed → fixed

↓ find pivot element in the 3-rd column

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \\ 0 & 0 & 0.04 & 0.03 & 0.12 \end{array}\right]$$

→ fixed → fixed

Perform 1 step of Gauss elim. to zero out 0.04: 1) $m_{43} = \frac{0.04}{100} =$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \\ 0 & 0 & 0 & -0.05 & -0.2 \end{array}\right]$$

2) $E_4 - m_{43} \cdot E_3 =$

$[0; 0; 0.04; 0.03 | 0.12] - [0; 0; \frac{0.04}{100} \cdot 100; \frac{0.04}{100} \cdot 200 | \frac{0.04}{100} \cdot 800]$

Obtain solution by backward substitution:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$-0.05 x_4 = -0.2 \Rightarrow x_4 = 4$

$100 x_3 + 200 x_4 = 800 \Rightarrow x_3 = 0$

$x_2 + 2x_3 + x_4 = 4 \Rightarrow x_2 = 0$

$x_1 + 2x_2 + x_3 + 1 \Rightarrow x_1 = 0$