MATHEMATICS
University of Gothenburg and Chalmers University of Technology
Examination in algebra : MMG 500 and MVE 150, 2017-06-08.
No books, written notes or any other aids are allowed.
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1a) Compute the product $\pi=(1\ 2)(2\ 3)(3\ 4)$ in S_4. 4p

b) Describe the permutations in the cyclic subgroup generated by π.

The permutations should be written in cycle form.

2 Let g,h be two elements in a finite group. Show that gh and hg have the same order. 4p

3.Determine the zero divisors and invertible elements in $\mathbf{Z}_{10}$. 4p

4 Let p be a prime. 5p
a) Show that the equation $x^p - 1 = 0$ has no other root than 1 in $\mathbf{Z}_p$.
b) Can the equation $x^p - a = 0$ have more than one root in $\mathbf{Z}_p$ for other elements $a \neq 1$ in $\mathbf{Z}_p$?

5. Let $* : G \times G \to G$ be an associative binary operation on a set G. 4p

a) Show that $(G, *)$ has at most one neutral element.

b) Show that each element of G has at most one inverse with respect to $*$.

6. Show that any finite integral domain is a field. 4p

All claims that are made must be motivated.