MATHEMATICS
Univ. of Gothenburg and Chalmers University of Technology
Examination in algebra : MMG500 and MVE 150, 2018-06-01.
No books, written notes or any other aids are allowed.
Telephone 031-772 5325.

1) Let ϕ be a homomorphism from the multiplicative group G to the additive group $\mathbf{Z}_2\times\mathbf{Z}_3\times\mathbf{Z}_4\times\mathbf{Z}_6$. Prove that $g^{12}\in\ker\phi$ for any $g\in G$. 3p

2). Prove that there is an element of order 990 in S_{30} . 4p

3) The center of a ring R is the subset $Z(R)$ of all elements x such that $xy = yx$ for all elements y in R. Show that $Z(R)$ is a subring. 4p

4) A bead is placed at each of the eight vertices of a cube, and each bead is to be painted either red or blue. Under equivalence relative to the group of rotations of the cube, how many distinguishable patterns are there? 4p

5. Formulate and prove Lagrange's theorem. 5p

(You may use general results from set theory, but any result on cosets that is needed should be proved.)

6, Show that a polynomial of degree $n\geq 1$ over a field F has at most n roots in F. 4p