MATHEMATICS
Univ. of Gothenburg and Chalmers University of Technology
Examination in algebra : MMG500 and MVE 150, 2018-03-16.
No aids are allowed. Telephone 031-772 5325.

1a) Let $\sigma=(123)$ and $\tau=(145)$. Compute the commutator $\sigma\tau\sigma^{-1}\tau^{-1}$ in S_5. 3p
(The answer should be given in cycle form.)
b) Show that $\sigma\tau\sigma^{-1}\tau^{-1}$ belongs to the subgroup A_5 of even cycles in S_5. 1p

2a) Let ϕ be a homomorphism from $\mathbf{Z}$ to a finite group G of order n. 3p
Prove that $\langle n\rangle \subseteq \ker \phi$.
b) Show that $\ker \phi = \langle n\rangle$ if and only if ϕ is surjective. 2p

3. Show that the rings $R= \mathbf{Z}[\sqrt{2}]=\{a+b\sqrt{2}: a,b\in\mathbf{Z}\}$ and 4p
$S=\left\{ \begin{pmatrix} a & 2b \\ b & a \end{pmatrix} : a,b\in\mathbf{Z}\right\}$ are isomorphic.

4a) Verify that $1/(3+2\sqrt{2})\in\mathbf{Z}[\sqrt{2}]$. 2p
b) Prove that $R=\mathbf{Z}[\sqrt{2}]$ has infinitely many units. 2p

5. Formulate and prove the fundamental homomorphism theorem for groups. 4p

6. Prove that any ideal of a polynomial ring $F[x]$ over a field F is a principal ideal. 4p

The theorems in Durbin's book may be used to solve exercises 1–4, *but all claims that are made must be motivated.*