Home assignment II - in ODE and Mathematical modeling.
Self excitable oscillations in electrical circuits. Lienard equation. Van der Pol equation.
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You are encouraged to work in small groups of 2-3 people, but each of you must write an own report (handwritten or in TEX) including: ***analytical work, theoretical argumentation, results and their interpretation after the plan below,*** *and names of the group members.* ***Send Matlab codes with clear comments by e-mail.*** *Deadline for the work is the 31-st of May. Grades for your reports will contribute 30% to the final marks. You are welcome to contact the teacher in his office or by e-mail to pose questions.*

1. Study the example about electrical circuits and Kirchhoff's laws on pages 76-77 in §3.3 and the continuation on pages 215-216 in §7.2 leading to Lienard's equation.

$$\dot{x} = y - f(x); \ \dot{y} = -x$$

2. Consider a variant of Van der Pol's equation, that is Lienard's equation 7.26 for $f(x) = \mu(\frac{1}{3}x^3 - x)$, $\mu > 0$. Solve Problem 7.7 p. 220.

3. Answer following theoretical questions. Is the fixed point in the origin for this system stable or unstable? Has this system a positively invariant set? You can answer the last question refering to the numerical results later if it seems to be difficult to do analytically.

4. Write a program in Matlab that solves Lienard's system of equations and illustrates solutions by orbits in the phase plane (as on pages 68-70 in the book) and by integral curves - graphs of $x(t)$ and $y(t)$ showing their dependence on time.

It might be useful to rescale variables y and t in the system so that non-linear term $f(x)$ will not include any parameters ($y/\mu \longmapsto y$, $t\mu \longmapsto t$).

5. Choose several values of μ : both small and very large. Draw phase portraits for the equation corresponding to these values of $\mu > 0$ together with the graph of the function $y = f(x)$. Draw pictures of integral curves $x(t)$ and $y(t)$ **.**

6. What types of orbits you observe? How they behave? Compare numerical pictures with your theoretical analysis above.

7. Try to explain the dynamics of the system for large μ.

8. Write a program that computes the Poincare map P for this system (read §6.4). Here Poincare map is a function P that maps a point y on the positive y - axis into another point $P(y)$ on the same positive axis in the following way. We compute a trajectory $\phi(t,(0,y))$ that starts in the point $(0,y)$, $y > 0$ and find y - coordinate $P(y)$ of the next intersextion point of on this trajextory $\phi(t,(0,y))$ with the positive y - axis. Stationary points $P(y) = y$ of this mapping correspond to periodic solutions of the original ODE.

9. Such Poincare map can be computed using the EVENT option in ODE45 in Matlab.

10. Draw a graph of the Poincare map togeter with the line $y = x$. Can one extract information about fixed points for P from this graph and information about periodic solutions to the ODE?

Extra question (not giving extra points!) for those who like to learn more theory.

Learn theory of Lienard's equation in §7.2 and compare it with your numerical results on the Poincare map.