Träningstenta i matematisk modellering, MMG511, MVE160

The examination includes two theorems with proofs and four problems of both theoretical and calculational character.

1. Formulate what is a generalised eigenspace of a matrix. Formulate and prove the theorem describing conditions implying that a solution to a linear autonomous system of ODE will tend to zero with time going to infinity or will be bounded for all times. **(4p)**

2. Formulate and give a proof for Liapunovs theorem on stability of a fixed point and give definitions of the notions used in the formulation. **(4p)**

3. **Linear systems**

   Consider the following ODE:

   $\dfrac{d\overrightarrow{r}(t)}{dt} = A\overrightarrow{r}(t),\ \overrightarrow{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix}$ with a constant matrix A defined as $A = 2I + C =$ $-2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$

   Calculate $\exp(At)$ and the general solution for the ODE. **(4p)**

4. Show that a solutions to the following system of equations exist on an infinite time interval.

   $$x' = \sin(xy) + 2x/\left(1 + x^2\right)$$
   $$y' = \ln(1 + x^2) + 3y$$

   (4p)

5. Consider the following system of ODE and investigate stability of the fixed point in the origin. **(4p)**

   $$\begin{cases} x' = x^3 + 2xy^2 \\ y' = \left(x^2 + 1\right) y \end{cases}$$

6. Show that the following system of ODE has a periodic solution. **(4p)**

   $$\begin{cases} x' = y \\ y' = -x + y(1 - 3x^2 - 2y^2) \end{cases}$$

   Max. 24 points;

   For GU: **VG:** 19 points; **G:** 12 points. For Chalmers: **5:** 21 points; **4:** 17 points; **3**: 12 points;

One must pass both the home assignments and the exam to pass the course.

Total points for the course will be the average of points for the home assignments (30%) and for this exam (70%)