Träningstenta i matematisk modellering, MMG511, MVE160

The examination includes two theorems with proofs and four problems of both theoretical and calculational character.

1. Formulate and give proof of the theorem on asymptotic stability of periodic linear systems of ODE. **(4p)**

2. Formulate and give a proof for Liapunovs theorem on instability of a fixed point and give definitions of the notions used in the formulation. **(4p)**

3. Consider the following ODE:

   $\frac{d\overrightarrow{r}(t)}{dt} = A\overrightarrow{r}(t),\ \overrightarrow{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}$ with $A = \begin{bmatrix} 1 & -4 \\ -1 & 1 \end{bmatrix}$

   Find the general solution for this system. **(2p)**

   Find which type has the stationary point at the origin and give a possibly exact sketch of the phase portrait. **(2p)**

4. Consider the following operator

   $$K(x)(t) = \frac{1}{4\pi}\int_0^{\pi} \cos(t-s)x(s)ds + t^2,$$

   acting in the Banach space $C([0,\pi])$ of continuous functions with norm $\|x\| = \sup_{t\in[0,\pi]} |x(t)|$.

   Show using Banach's contraction principle that $K(x)(t)$ has a fixed point. **(4p)**

5. Consider the system of equations: $\begin{cases} x' = yx - 5x^3 \\ y' = x^2 - 5y \end{cases}$ **(4p)**

   Investigate if the origin is an asymptotically stable stationary point.

6. Use Bendixsons theorem to show that the system of equations.

   $\begin{cases} x' = 3xy^2 - x^2y \\ y' = 5x^2y - xy^2 \end{cases}$

   does not have periodical solutions. **(4p)**

   Max. 24 points;

   For GU: **VG:** 19 points; **G:** 12 points. For Chalmers: **5:** 21 points; **4:** 17 points; **3**: 12 points;

One must pass both the home assignments and the exam to pass the course.

Total points for the course will be the average of points for the home assignments (30%) and for this exam (70%)