ODE and modeling MMG511-detailed planning.

Components: theorems, important examples, theoretical problems, typical applied problems.

Introduction to ideas and methods via 1-d analysis

1. §1.1, 1.2 Notions, notations. Classification of ODE: order, autonomous, linear, non-linear.
2. Examples from real life: Newton, mass action law, Ohm's law …
3. §1.3. Elementary 1-d examples and exercises.
   Problem 1.11 – separable variables. Problem 1.16 – Example of logistic growth without and with optimal harvest – at the lecture.
4. §1.4. Explicit solutions. Linear equation the only taken in the class – (other - for home work)
5. §1.5. Qualitative analysis in 1-d: autonomous case. Example - optimal harvest.
   DO NOT TAKE §1.5. Qualitative analysis in 1-d: NON-autonomous case.
   DO NOT TAKE §1.5. sub and super-solutions. Lemma 1.2
6. §1.5. Lipschitz continuity. Uniquness in 1-d. Theorem 1.3.
7. §1.6. Qualitative analysis of periodic solutions. Poincare' map. Example of logistic growth with periodic harvest. (typical problem)
8. Problems 1.13-1.17 – partially as demo in the class and partially as exercises.
9. Problems 1.27, 1.28, 1.32-1.36

Linear systems with constant coefficients

10. §3.1 Matrix exponential from Picard iterations.
11. §3.1 Matrix norm. exponent properties Lemma 3.1. Linear change or variables.
12. §3.1 Jordan form for complex matrices.Theorem 3.2. (Correct ???) exponential of a Jordan form matrix. Exponential of an arbitrary 2x2 matrix.
13. §3.2 2-d linear systems in plane: all cases with phase portraits (Problem 3.14).
14. §3.2 Stability of linear systems in general case. Theorem 3.4, Corollary 3.5. Corollary 3.6.
15. §3.2 Duhamel formula.3.48
16. §3.2 Problems of the type 3.9, 3.10.
17. DO NOT TAKE §3.3 Linear autonomous equations of order n (it is an elementary topic studied in Calculus)
18. Electrical circuit. – nonhomogeneous problem. Resonanses.
19. Problems 3.12, 3.13, 3.18 -resonanses.

General Linear systems

20. §3.4 General linear system (time dependent). Superposition principle. Principle

matrix solution. Theorem 3.10. Principal matrix solution. Fundamental matrix solution. Wronsky determinant.

~~Existence and uniqueness on infinite interval – Arnold p.168~~

21. General inhomogeneous linear equation.Theorem 3.12.
22. DO NOT TAKE d'Alembert reduction. Problems 3.27,3.28, 3.30,

Periodic linear systems

23. §3.6. Periodic linear systems. Floquet's theory. Theorem 3.15.
24. Hill equation. Theorem 3.19
25. Home assignment 1. Linear theory of inverted pendulum. Mathieu equation. Stability diagram
26. Problems 3.39, 3.40, 3.41, 3.42, 3.43
27. WE MIGHT NOT TAKE §3.7 Perturbed systems stability. Stability by linearization. 1-d example. Theorem 3.20, Theorem 3.23, Theorem 3.26, Corollary 3.27…

Existence-uniquness- data variation

---

28. §2.1 Banach spaces, fixed point theorems: Theorem 2.1 (Banach contraction principle), (Bohl-Brower?? Fixed point theorem) Problems 2.1, 2.2, 2.3, 2.4
29. §2.2 Solution notions. Existence Theorem 2.2. (Picard-Lindelöf). Problems 2.5, 2.6, 2.7, 2.8.
30. §2.3 Extensions of the existence theorem. Theorem 2.4 (Weissinger criterium for a fixed point).

Alternative proof to theorem 2.2 similar to the proof of Theorem 2.5. WE DO NOT TAKE THEOREM 2.5 ITSELF. Problems 2.9, 2.10

31. §2.4 Dependence on initial conditions. Theorem 2.8. Simpler version of Gronwall's inequality formula 2.38
32. §2.6 Extensibility of solutions, maximal solution, global solution. Lemma 2.14. Corollary 2.14, Corollary 2.15, Theorem 2.17.
33. WE WILL NOT TAKE §2.7 Eulers metod. Peano theorem. Arzela Ascoli Theorem.
34. Chapter 2. Problems 2.18, 2.20.

Stability, limit sets

35. Stability of solutions and fixed points by linearization §3.7. Theorem 3.26, Corollary 3.27.,
36. Exercises on stability by linearization
37. §6.2 , 6.3 Flows of autonomous equations. Notions of orbits, maximal integral curves, invariant sets, limit sets.
38. WE DO NOT TAKE §6.4
39. §6.5, 6.6 Stability of fixed points. Liapunovs functions. Liapunovs theory.

40.Exercises on Liapunovs theory.

Planar dynamical systems

41. Ecology: Volterra Lotka, limited growth, competing species, Van der Pol equation.
42. Poincare Bendixson theorem.(without proof)

Bifurcations

43. Bifurcation examples. Puincare-Andronov-Hopf bifurcation. Brusselator-example.
44. Exercises.