Third Exercise Session: 7/5

Theme: Graph Theory 1 (degrees, isomorphism, planarity, paths and cycles, vertex coloring)

Relevant Chapters: 15, 16

1. Consider the four sequences

$1, 2, 3, 3, 3, 4, 4, 5$
$1, 2, 4, 4, 4, 7, 7, 7$
$3, 4, 4, 4, 4, 7, 7, 7$
$1, 2, 3, 3, 3, 3, 4, 5.$

The first two are not *graphic* (see Exercise 15.8.17 for the definition of this word), whereas the last two are. Prove these assertions.

2. (6.47-6.49 in EG-1) Which pairs of graphs in Figure O.3.2 are isomorphiic ? Motivate your answers well !

3. (i) Determine an Euler cycle in Figure O.3.3(i) which begins and ends at node A.
(ii) Determine a route for Dr. Dodder in Exercise 15.4.4 in Biggs.

4. (7.13 in EG-2) A *Platonic solid*[1] is a regular convex polyhedron, i.e.: it is convex and all of its faces are regular polygons with the same number of edges and the same angles. A Platonic solid is therefore determined by a triple of numbers (n, e, d), where n is the number of polygonal faces, e is the number of edges on each face and d is the degree of every vertex.

It is known that there are exactly five Platonic solids, given in the following table:

Name	n	e	d
Tetrahedron	4	3	3
Cube	6	4	3
Octahedron	8	3	4
Dodecahedron	12	5	3
Ikosaheder	20	3	5

Using Euler's theorem on planar graphs, prove that there are no other Platonic solids.

5. For each of the graphs in Figure 11.84, determine whether or not it has a Hamilton path (resp. Hamilton cycle). Motivate your answers well !

[1] See https://en.wikipedia.org/wiki/Platonic_solid.

6. A *tournament* on n nodes is a directed K_n, i.e.: a complete graph where every edge has been assigned a direction.

(i) Prove that every tournament has a Hamilton path.
(ii) Give an example of a tournament with a Hamilton cycle.

7. **(i)** For each graph G in Figure O.3.7, determine its chromatic number along with an explicit vertex $\chi(G)$-coloring.
(ii) For the bottom graph, determine an ordering of the vertices for which the greedy algorithm would use 5 colors.