Fourth Exercise Session: 21/5

Theme: Graph Theory 2 (Trees, matchings, flows)

Relevant Chapters: 16, 17, 18

1. For the graph in Figure O.4.1,

(i) Apply both Prim's och Kruskal's algorithm to determine a minimal spanning tree. In each case, list the edges which the algorithm chooses at each step.

(ii) Apply Dijkstra's algorithm to determine a shortest path from s to t. List the edges chosen and the labels assigned at each step.

2. (9.1(a) in EG-2) Consider the graph in Figure O.4.2, with the indicated matching. Apply the augmenting-path algorithm three times to produce a perfect matching.

3. (9.2(a) in EG-2) Prove that the graph in Figure O.4.3 has no perfect matching.

4. (Ex. 9.2 in EG-2) Skruttemåla Council wish to choose an Environmental Advisory Panel. The panel should contain at most one representative from each of the local lobby groups. Each such group has a steering committee consisting of three members (chairman, secretary and treasurer). There are seven groups, with the following committee members:

Växternas vänner: Axel, Berta, Cecilia.
Naturens nördar: Daniela, Axel, Berta.
Djurens djupingar: Axel, Cecilia, Daniela.
Sjöarnas skönandar: Axel, Erik, Frippe.
Korallens kompisar: Frippe, Gerd, Henrik.
Pörtenas partner: Cecilia, Berta, Daniela.
Strändernas stöd: Cecilia, Axel, Berta.

How many groups can be represented in the Panel ? Determine such a Panel of maximum size.

5. (9.30 in EG-2) Stefan has invited in Ulf, Jimmie, Annie and Jonas to eat fruit salad and weld together a blocköverskridande agreement on the epochal question of whether ballot papers in September should be 10 x 12 cm or 12 x 15 cm. Six different ingredients are available. Stefan lets his imagination run wild and makes up five salad bowls, comprising the following mixtures:

- apples, bananas, oranges, melons and nuts
- bananas, oranges and raisins
- bananas, oranges, melons, nuts and raisins
- bananas, raisins, melons and nuts

- apples, bananas, oranges and nuts.

When the guests arrive it turns out, however, that Ulf doesn't like nuts, that Jimmie can't stand oranges, that Annie is allergic to apples, and that Jonas can't imagine anything worse than eating bananas and raisins together (except, perhaps, vinster i välfärden). Stefan will, however, eat anything.

Describe a way to feed as many people as people, i.e.: draw and find a maximum matching in an appropriate bipartite graph.

6. (9.33 in EG-2) (a) How many perfect matchings are there in the cycle C_n ?

(b) How many perfect mathcings are there in P_n, the chain with n vertices ?

(c) Let a_n be the number of perfect matchings in the graph in Figure O.4.6. Prove that $a_{n+2} = a_{n+1} + a_n \; \forall \, n \geq 1$ and thereby give a formula for a_n.

7. (8.8-8.9 in EG-2) Start with the everywhere-zero flow and apply the Ford-Fulkerson algorithm to determine a maximum strength flow in the network of Figur O.4.7. Indicate which augmenting path you choose at each step. Determine a corresponding minimum capacity cut.