Exercises III

29/3 2012

Due 16/4

1 Find all the solutions x modulo n such that $x^2 = 1(n)$ with $n = 25$ or $n = 35$

2 if $p = 3(4)$ show that there is no integer n such that $x|n^2 + 1$

3 'Explain' why $101, 103, 107, 109$ are all primes. Hint $3 \times 5 \times 7 = 105$

4 Determine the fraction that corresponds to $LRRLRR$.

5 Find the 'address' for $\frac{17}{29}$

6 Given x corresponding to a string S what fraction does S^* correspond to, where S^* is the string gotten from S by interchanging L for R and R for L

7 By the height of a fraction $\frac{m}{n}$ with $m, n \geq 0$ is meant the number $m + n$.
i) Give the list of all fractions with height 5
ii) Given a sequence S of length n give an upper and lower bound for its height in terms of n, and try to make them as sharp as possible.

8 Consider the Fermat numbers $3, 5, 17, 257, 65537..$ is it true that all the remaining Fermat numbers end with 7?

9 Show that 641 divides $F_5 = 2^{2^5} + 1$ by using the facts that $641 = 2^4 + 5^4$ and $641 = 5 \times 2^7 + 1$

10 Show that any two distinct Fermat numbers F_n and F_m are relatively prime, and conclude that there exists infinitely many primes.

11 Show that the sum of the infinite series $\sum_{n>0} \frac{n}{3^n}$ is a rational number

12 Given $x \in (0, 1)$ and let $0.b_1b_2\ldots$ be its binary expansion. Replace each occurence of 0 with L and each occurence of 1 with R and consider the corresponding number $\phi(x)$. Show that ϕ is a strictly increasing function. Can you compute its derivative at different points x.

13 Show that $RLRLRL\ldots$ satisfies a quadratic equation. Hint: If x corresponds to S what does RLS correspond to?