Exercises V

May 9 2012

May 14

1 Write down closed forms for the generating functions of the following series

i)

$$\binom{r}{n}x^n$$

ii)

$$n+1$$

iii)

$$\frac{1}{n+1}$$

iv)

$$\{ {n \atop m} \} \quad (m \quad \text{fixed})$$

v)

$$[{m \atop n}] \quad (m \quad \text{fixed})$$

vi)

$$\{ {n \atop m} \} \frac{m!}{n!} \quad (m \quad \text{fixed})$$

2 A series g_n is given recursively by $g_0 = 1$ and

$$g_n = g_{n-1} + 2g_{n-2} + \ldots ng_0$$

Find an explicit formula.

3 In how many ways could you change 25 'öre' in the old times whene there were coines with denominations $1, 2, 5, 10$ and 25 'öre'?

4 Find an explicit formula in terms of n for the sums $\sum_{a+b+c=n} abc$ where a, b, c are non-negative integers.

5 Consider the function $B_3(z) = \sum_{n\geq 0} \binom{3n+1}{n} \frac{1}{3n+1} z^n$
i) Show that $B_3(t)$ is positive and strictly increasing on the positive real axis
ii) Use this function to solve a cubic equation $x^3 + px + q$