Tentamensskrivning

MMG610

Diskret Matematik

29/5 2010

1 [5] In how many ways can we permute the letters in the 'word' AABBBCCCCD?

2 [5] Compute $1001^{1000001}$ modulo 19

3 [5] Compute the Eulerfunction $\phi(n)$ för $n = 17017$

4 [5] Compute the following Stirling numbers $\{ {12 \atop 4} \}$ and $[{12 \atop 5}]$

5 [10] Compute F_{1000} modulo 1001, where F_n denote the Fibonacci numbers.

6 [10] Find the maximum of all integers n such that 10^n divides 10^6!

7 [10] Find the closed form of the sum $S_4(n) = \sum_{k=0}^{n-1} k^4$

8 Define the numbers C_n by $\sum_{n=0}^{\infty} C_n z^n = \frac{1}{e^z+1}$
a) [5] Find $C_0, \ldots C_4$
b) [10] Find a relation between C_n and the Bernoulli numbers B_n

9 Define the functions $P_n(x)$ by $\frac{P_n(x)}{(1+x^2)^{n+1}} = \frac{d^n}{dx^n} \frac{1}{1+x^2}$
a) [5] Set up a recursive formula for P_n and in particular prove that they are polynomials of degree n.
b) [5] Compute the values $P_n(0)$
c) [10] Try to get a closed form for the polynomials $P_n(x)$

10 [15] Consider the binomial coefficients $\binom{1000000}{k}$ for $0 \leq k \leq 1000000$. Compute the number of even ones!

Hand calculators are permitted.

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50 points will be sufficient for a passing grade. 80 points will be sufficient for passing with distinction.