EXERCISES: CHAPTER 16

Section 16.3 (Spanning trees and the MST problem)

1. Find spanning trees for the cube graph and for Petersen's graph.

2. Sketch all the 16 spanning trees for K_4.

3. Use the greedy algorithm to find a minimum spanning tree for the weighted graph in Figure 16.3.3. Is the minimum spanning tree unique in this case ?

4. Let G be the weighted graph whose vertex set is $\{x, a, b, c, d, e, f\}$ and whose edges and weights are given by the table

xa	xb	xc	xd	xe	xf	ab	bc	cd	de	ef	fa
6	3	2	4	3	7	6	2	3	1	8	6

Find all minimum spanning trees for G.

5. Suppose that T is a minimum spanning tree in a weighted graph K and e^* is an edge of K not in T. Let e be any edge of T belonging to the unique path in T which joins the vertices of e^*. Show that $w(e) \leq w(e^*)$.

Section 16.6 (The shortest path problem)

1. Use Dijkstra's algorithm to find the shortest path from v to w in the weighted graph depicted in Figure 16.6.1.

2. Find the shortest route from A to F in the weighted graph specified in the following table:

	A	B	C	D	E	F
A	–	5	8	3	4	9
B		–	6	1	5	4
C			–	3	9	2
D				–	4	6
E					–	3

Review Section 16.7

6. Find all minimum spanning trees for the weighted graph depicted in Figure 16.7.6.

8. In the weighted graph of Exercise 16.3.3, find the shortest path from a to k.

9. Kruskal's version of the greedy algorithm is: choose the edges in order of increasing weight, randomly in the case of ties, but rejecting those whose inclusion would create a cycle. Show by example that in Kruskal's method the set of edges constructed at an intermediate stage need not form a connected graph.

10. Prove that Kruskal's method always produces a MST.

17. Let the vertices of the complete graph K_n be denoted by $1, 2, \ldots, n$, and for each spanning tree T of K_n define the ***Prüfer symbol*** $(p_1, p_2, \ldots, p_{n-2})$ as follows: The Prüfer symbol of a tree with two vertices is null. If $n > 2$, the Prüfer symbol of a tree with n vertices is $(j, q_1, \ldots, q_{n-3})$ where

(i) j is the unique vertex of T adjacent to the vertex i of degree one which comes first in numerical order,
(ii) $(q_1, \ldots, q_{n-3})$ is the Prüfer symbol of the tree obtained by deleting the edge $\{i, j\}$ from T.

Show that the Prüfer symbol construction defines a bijection from the set of spanning trees of K_n to the set of ordered $(n-2)$-tuples from the set $\{1, 2, \ldots, n\}$ and deduce that K_n has n^{n-2} spanning trees.