MATEMATIK
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Exam, Analytic function theory, MMG700

Friday, August 24, 2018, $14^{00} - 18^{00}$

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1. Show that if $f\colon D(0,1) \to \mathbb{C}$ is complex differentiable at $a \in D(0,1)$ then f satisfies the Cauchy-Riemann equations at a. (3p)

2. Show that if f is holomorphic in a neighborhood of $\overline{D(0,1)}$ and $f(z) \neq 0$ for $|z| = 1$, then $\frac{1}{2\pi i}\int_{|z|=1} \frac{f'(z)\,dz}{f(z)}$ equals the number of zeros of f in $D(0,1)$ with multiplicity. (This is the Argument principle.) (3p)

3. (a) State Cauchy's residue theorem. (1p)

   (b) Compute the integral $\int_{|z|=4} \frac{z\,dz}{\sin z}$. (2p)

4. How many zeros does $4z^4 + 1 + e^z$ have in the disc $D(0,1)$? (3p)

5. Compute the Laurent expansion of $\frac{1}{z^2 - i\sqrt{2}z - 1}$ in $\{z;\, |z| > 1\}$. (3p)

6. Map the set $\{z;\, |z - i| < 1,\, \operatorname{Re} z > 0\}$ conformally onto the upper half-plane. (3p)

7. Compute the integral $\int_{-\infty}^{\infty} \frac{e^{-ixt}\,dx}{x^2+1}$ for all $t \in \mathbb{R}$. (3p)

8. Let f be a holomorphic function in the punctured disc $D(0,1) \setminus \{0\}$. Show that if $\operatorname{Re} f > 1$ then $1/f(z)$ is holomorphic in the disc $D(0,1)$. (3p)

Lycka till! /H.S.K.