MATEMATIK
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Exam, Analytic function theory, MMG700

Wednesday, January 3, 2018, $14^{00} - 18^{00}$

1. Show that if f is holomorphic in the disc $D(a, R)$ and $|f(z)| \le |f(a)|$ for all $z \in D(a, R)$, then f is constant. (This is the local maximum principle.) (3p)

2. State and prove the Open mapping theorem. (3p)

3. Let $\varphi(z) = \dfrac{z}{z+1}$.

   (a) Show that φ is conformal. (1p)

   (b) Compute the image under φ of the real axis and the unit circle. (1p)

   (c) Show that any conformal map preserves angles between curves. (1p)

4. Find the number of zeros of $z^5 + 4z^2 + z + 1$ in the annulus $\{z;\, 1 < |z| < 2\}$. (3p)

5. Find the Laurent series of $\dfrac{1}{(z-1)(z-i)}$ in $\{z;\ |z| > 1\}$. (3p)

6. Classify the singularities of $f(z) := \dfrac{\tan(z/2)}{z^2(1+z^2)}$ and compute the integral $\displaystyle\int_{|z|=2} f(z)\, dz$. (3p)

7. Compute $\displaystyle\int_0^\infty \frac{\cos(ax)}{1+x^2}\, dx$ for all $a \in \mathbb{R}$. (3p)

8. Let u be a real-valued harmonic function in $\mathbb{C}$. Show that $(u'_x)^2 - (u'_y)^2$ and $u'_x u'_y$ are harmonic. (3p)

Lycka till! /H.S.K.