MATEMATIK
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Exam, Analytic function theory, MMG700

Friday, October 27, 2017, $8^{30} - 12^{30}$

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1. Show that if $f = u + iv$ is complex differentiable at a, then u and v satisfy the Cauchy-Riemann equations at a. (3p)

2. Let Ω be a bounded open set whose boundary, $\partial\Omega$, is a smooth curve. Show that if f is holomorphic in an open set containing $\overline{\Omega}$ and $f(z) \neq 0$ for $z \in \partial\Omega$, then $\frac{1}{2\pi i}\int_{\partial\Omega}\frac{f'(z)\,dz}{f(z)}$ is the number of zeros of f in Ω counted with multiplicity. (3p)

3. (a) Show that if f is holomorphic in an open set Ω and $\mathcal{C}$ is an oriented smooth curve in Ω starting at a and ending at b, then $\int_{\mathcal{C}} f'(z)\,dz = f(b) - f(a)$. (2p)

   (b) Compute the integral $\int_{\mathcal{C}}\frac{dz}{(z-1)^2}$, where $\mathcal{C}$ is the straight line starting at 0 and ending at $1+i$. (1p)

4. How many zeros (counting multiplicity) does $z^4 + z^3 + 5$ have in the annulus $\{z;\ 1 < |z| < 2\}$? (3p)

5. Compute the integral $\int_0^\infty \frac{\cos 2x}{(x^2+1)^2}$. (3p)

6. Let $\varphi(z) = \frac{1}{z - i\sqrt{2}}$.

   (a) Find the fixed points of φ. (A fixed point is a point a such that $\varphi(a) = a$.) (1p)

   (b) Find the image under φ of the lower half of the disc $\{z;\ |z - i/\sqrt{2}| < 1/\sqrt{2}\}$. (2p)

7. Find a conformal map of the set $\{z;\ 0 < \arg z < \pi/3, |z| < 1\}$ onto the unit disc. (3p)

8. Let f be holomorphic in $\mathbb{C}$. Show that $\frac{1}{2\pi i}\int_{\partial D(0,1)}\overline{f(z)}\,dz = \overline{f'(0)}$. (3p)

Good luck! /H.S.K.