Home work. Part 1[1].

1. (a) Find the Fourier series for the 2π-periodic functions $f(x) = \sin^3 x$ and $g(x) = \cos^3 x$ and compute then $\int_0^{2\pi} f(x)^2 dx$.

(b) Find the Fourier series for $\sin^n x$ and $\cos^n x$ ($n \geq 1$ arbitrary integer) (Hint: Use binomial formula and Euler's formula. The expansion depends on n being even or odd.)

(c) Apply Parseval's formula to your result in (b) and rewrite the sum $\sum_{k=0}^{n} \binom{n}{k}^2$ as an integration of the function $\cos^{2n} x$ on $(-\pi, \pi)$

2. Are the following series point-wise convergent? If so, how smooth are the functions, i.e., are they in C^0, C^1, ..., or C^∞?

$$(a) \quad f(\theta) = \sum_{n=1}^{\infty} \frac{1}{n+1} \sin n\theta$$

$$(b) \quad f(\theta) = \sum_{n=1}^{\infty} \frac{1}{n^3} \sin n\theta$$

$$(c) \quad f(\theta) = \sum_{n=0}^{\infty} \frac{1}{3^n} \cos n\theta$$

[1]The deadline for submitting your solutions is Thurs. Sept 14. You may work together in teams of max 3 members each, and you need only submit your team's solutions