

## Home work. Part 2<sup>1</sup>.

1. (a) Find the sum

$$\sum_{n=1}^{\infty} \frac{1}{n} e^{in\theta}$$

(by using Taylor expansion of some known function - write  $z = e^{i\theta}$  check the table of Taylor expansions) and prove that the series is  $L^2(-\pi, \pi)$ -convergent but not point-wise convergent.

(b) For which  $p > 0$  is the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} e^{in\theta}$$

$L^2$ -convergent?

(c) For which  $p > 0$  is the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} e^{in\theta}$$

uniformly convergent on  $\mathbb{R}$ ?

2. Which of the following Sturm-Liouville problems on  $[0, \pi]$  is regular? Solve then the regular problem

(a)  $f'' + \lambda f = 0$ ,  $f(0) = 0$ ,  $f'(\pi) = 0$ .

(b)  $f'' + \lambda f = 0$ ,  $f(0) = 0$ ,  $f'(\pi) = 0$ .

(c)  $f'' + \lambda f = 0$ ,  $f'(0) = 0$ ,  $f(\pi) = 0$ . (Change variables  $x \rightarrow \pi - x$  and use (b))

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<sup>1</sup>The deadline for submitting this home work is Thursday, Sept 28