

Home work. Part 3¹.

1. (a) Compute the Fourier transforms of the functions

$$f(x) = x^2 e^{-x^2}, g(x) = \frac{x}{(1+x^2)^2}$$

- (b) Find the integral $\int_{-\infty}^{\infty} |g(x)|^2 dx$.

- (c) Are f and g in the space $L^1(\mathbb{R})$?

2. (a) Find the Laplace transforms of the following functions

$$f(t) = t^3 \sin t, g(t) = t \sinh^2 t$$

(Hint: Use “double-angle-formula” for $\sinh t$.)

- (b) Find the inverse Laplace of the following functions

$$\frac{1}{z^2 + 2z + 1}, \quad \frac{1}{z^2 + 2z - 3}, \quad \frac{1}{z^2 + z + 1}.$$

(c) You might have observed that Laplace transforms of many familiar functions are rational functions. Prove the following general claim: Let $p(t)$ be a polynomial. Then the Laplace transform $F(z) = \mathcal{L}[f](z)$ of $f(t) = p(t)e^{ct}$ is a rational function of z . (Recall that $F(z)$ is called rational if $F(z) = \frac{P(z)}{Q(z)}$ where P and Q are polynomials)

¹The deadline for submitting this home work is Wednesday Oct. 11, 2017