Fourier analysis fall 2008. Exercises 3.

1. Exercise 1.3.1 in Folland.

2. Exercise 1.3.7 in Folland.

3. Consider the vibrating string on $0 \leq x \leq \pi$ as in the lecture. Supppose that the initial conditions are $u(x,0) = \sin(2x)$, $u'_t(x,0) = 3\sin(5x)$. What is the solution $u(x,t)$?

4. Check the following entries in Table 1 in Folland: Entry 2, 4, 6, 17. What is, according to Theorem 2.1, the sum of the Fourier series?

5. If you know the Fourier coefficients of f, what can you say about the Fourier coefficients of $f(x-a)$ and of $e^{ikx}f(x)$ (where a is real and k is integer)? Compare with the shift rules for Laplace transform.

6. When f and g are 2π-periodic Riemann integrable functions, define their convolution by

$$(f * g)(x) = \frac{1}{2\pi}\int_0^{2\pi} f(y)g(x-y)\,dy.$$

   Denoting Fourier coefficients by $c_n(f)$, show that $c_n(f*g) = c_n(f)c_n(g)$.

7. Let f be the 2π-periodic function defined by $f(x) = x$ for $-\pi < x \leq \pi$. As I showed in the lecture, it has Fourier coefficients $c_n = (-1)^n i/n$ $(n \neq 0)$, $c_0 = 0$. What does Bessel's inequality say for this function? Do we in fact have equality?

Answer to question 3: $u(x,t) = \sin(2x)\cos(2ct) + \dfrac{3}{5c}\sin(5x)\sin(5ct)$.