Fourier analysis fall 2008. Exercises 8.

1. Compute the Fourier transform of

   (a) $e^{-|x|}$,

   (b) $x\,\chi_{[-1,1]}(x)$,

   (c) $\sin x\,\chi_{[-\pi,\pi]}(x)$,

   (d) $e^{-x}H(x)$,

   (e) $e^{-|x|}\cos(x)$.

   Here, H is Heaviside's function and $\chi_{[a,b]}(x)$ is the characteristic function of $[a,b]$, that is, $\chi_{[a,b]}(x) = 1$ for $x \in [a,b]$ and 0 else.

2. If $f(x)$ has Fourier transform $\hat{f}(\xi)$, what is then the Fourier transform of $\cos(x)f(2x+1)$?

3. Complete the proof of (a) and (b) in Folland, Theorem 7.5.

4. Folland, Exercise 7.3.1.

Answers:

1. (a) $\dfrac{2}{\xi^2+1}$, (b) $2i\dfrac{\xi\cos\xi - \sin\xi}{\xi^2}$, (c) $\dfrac{2i\sin(\pi\xi)}{\xi^2-1}$, (d) $\dfrac{1}{1+i\xi}$, (e) $\dfrac{2(\xi^2+2)}{\xi^4+4}$.

2. $\dfrac{1}{4}\left(e^{\frac{1}{2}i(\xi-1)}\hat{f}\left(\dfrac{\xi-1}{2}\right) + e^{\frac{1}{2}i(\xi+1)}\hat{f}\left(\dfrac{\xi+1}{2}\right)\right)$.