Fourier analysis fall 2008. Check-list for theory.

The complete course literature is as follows.

- Asadzadeh: Chapter 1.
- Notes 1 on Laplace transform.
- Folland: Chapter 1.3, 2.1–2.5 (you may skip Theorems 2.4 and 2.6), 4.1–4.4, 7.2 (skip the Fourier Inversion Theorem on p. 218, we have only discussed Theorem 7.6), 7.3 (section on PDE), 7.6 (section on discrete Fourier transform, but not fast Fourier transform).
- Folland Chapter 3: Section 3.1 should in principle be known from linear algebra. Sections 3.2–3.4 can be replaced by Notes 4. The "dominated convergence theorem" on p. 83 is not included. In Section 3.5–3.6 everything up to Thm. 3.9(b) is included, the rest can be viewed as supplementary (you need not know Thm. 3.9(c) or Thm. 3.10 at the exam).
- Notes 3: You should know the technique for finding periodic solutions of ODEs as Fourier series, but the discussion of the Gibbs phenomenon is supplementary.

Notes 2 (Bessel's inequality) is contained in Notes 4. Notes 5 (filters) is supplementary.

Here is a check-list that you can use to see if you have understood the theory.

- What is a piecewise continuous function? What is a piecewise smooth (or C^1) function?
- What is Laplace transform?
- What is the Laplace transform of $f'(x)$ and $xf(x)$? What is the first and second shift rule?
- What is convolution? Why is it commutative? Why is it associative?
- What is the Laplace transform of a convolution?
- Formulate a uniqueness theorem for the Laplace transform. Show that it follows from the inversion formula for Fourier transform (see Folland, page 266–267).
- What is the fundamental solution (or impulse response) of an ODE (linear, with constant coefficients)? How can the solution to an inhomogeneous equation be obtained from the fundamental solution?
- What is the Fourier series of a periodic function? How does one pass between complex and real form of a Fourier series? What is the Fourier cosine and Fourier sine series for functions defined on an interval $[0, T]$?
- Formulate and prove the Riemann–Lebesgue lemma for Fourier series.
- What is the Dirichlet kernel and what role does it play in the theory of Fourier series?

- Formulate and prove the inversion formula for Fourier series (Folland, Thm. 2.1).
- Formulate and prove a statement on differentiation of Fourier series (Folland, Thm. 2.2).
- Formulate and prove a statement on uniform convergence of Fourier series (Folland, Thm. 2.5).
- What is an inner product on a complex vector space?
- What is a Hilbert space?
- What, at least roughly, do we mean by $L^2([a,b])$ and $L^2(\mathbb{R})$?
- What is meant by convergence in norm, for "abstract" spaces and for L^2-spaces?
- What is the relation between the following three: Convergence in $L^2([a,b])$, uniform convergence on $[a,b]$, pointwise convergence on $[a,b]$?
- Formulate and prove Bessel's inequality on general inner product spaces.
- What is a complete orthogonal system? Why are the two definitions given in Notes 4 equivalent?
- Which complete orthogonal system is related to Fourier series? Why is it complete (sketch of proof)? What does this mean (norm convergence, Parseval's formula)? What about Fourier sine and cosine series?
- What is the best way to approximate a vector in norm with linear combinations of vectors in a finite orthogonal system? What does this tell us about partial sums of Fourier series?
- What is Fourier transform? What is the Fourier transform of $f'(x)$, $xf(x)$, $f(x+a)$, $e^{iax}f(x)$? What is the Fourier transform of a convolution?
- Formulate the Riemann–Lebesgue lemma for Fourier transform. Sketch the proof, for Riemann integrable functions.
- Prove an inversion theorem for the Fourier transform (Folland, Thm. 7.6).
- What is the Plancherel theorem for Fourier transform (proof not included)?
- What is finite (or discrete) Fourier transform? Prove the inversion formula.
- What is a symmetric operator? What can you say about eigenvalues and eigenvectors (Folland, Thm. 3.9(a),(b))? (On the lecture we did this on general inner product spaces, Folland does it only for second order ordinary differential operator with boundary conditions.)
- Give some criteria that guarantee that a second order ordinary differential operator, with boundary conditions, is symmetric.
- Derive Poisson's integral formula for a disc, and for a half-plane.