

# Fourier analysis (MMG710/TMA362)

**Time:** 2010-01-09, 08.30–13.30

**Tools:** No calculator or handbook is allowed

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**Grades:** Each problem gives 4 points. For MMG710 grades are G (12-17 points) and VG (18-24 points). For TMA362 grades are 3 (12-14 points), 4 (15-17 points) and 5 (18-24 points).

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- 1 Compute the Laplace transform of the function

$$f(x) = \begin{cases} x - x^2, & 0 \leq x \leq 1, \\ 0, & x > 1. \end{cases}$$

- 2 Find numbers  $a_n$  such that

$$x^2 = \sum_{n=1}^{\infty} a_n \sin(nx), \quad 0 < x < \pi.$$

What is the value of the series

$$\sum_{n=1}^{\infty} a_n \sin(4n)?$$

- 3 Solve (as an infinite series) the problem

$$\begin{cases} u'_t = u''_{xx}, & t > 0, \quad 0 < x < \pi, \\ u'_x(0, t) = u'_x(\pi, t) = 0, & t > 0, \\ u(x, 0) = 1, & 0 < x < \pi/2, \\ u(x, 0) = 0, & \pi/2 < x < \pi. \end{cases}$$

- 4 Find numbers  $a$  and  $b$  such that the integral

$$\int_0^{2\pi} |e^x - ae^{ix} - be^{-ix}|^2 dx$$

is minimized. Also compute the minimal value of the integral.

- 5 Let  $g_a(x) = 1/(x^2 + a^2)$ . Compute

$$\int_{-\infty}^{\infty} |(g_a * g_b)(x)|^2 dx.$$

You may use that  $1/(x^2 + 1)$  has Fourier transform  $\pi e^{-|\xi|}$ .

- 6 Formulate and prove the inversion theorem for Fourier transform.

**Good luck!**

Hjalmar