

Fourier analysis (MMG710/TMA362)

Time: 2010-08-23, 08.30–13.30

Tools: No calculator or handbook is allowed

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Grades: For MMG710 grades are G (12-17 points) and VG (18-24 points). For TMA362 grades are 3 (12-14 points), 4 (15-17 points) and 5 (18-24 points).

- 1 Use Laplace transform to solve the initial value problem

$$x''(t) - 2x'(t) + x(t) = e^t, \quad x(0) = 0, \quad x'(0) = 1. \quad (4p)$$

- 2 In standard tables, the following identity can be found:

$$x(\pi - x) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin((2n-1)x)}{(2n-1)^3}, \quad 0 < x < \pi. \quad (\star)$$

- (a) Prove the identity (\star) . (3p)

- (b) Use (\star) to compute the sum

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}. \quad (2p)$$

- (c) Formulate and solve a boundary value problem for one of the classical partial differential equations of mathematical physics. The identity (\star) should be used in the solution. (3p)

- 3 (a) Define the notion of convergence in $L^2([0, 1])$. (2p)

- (b) Give an example of a sequence of functions $(f_n)_{n=1}^{\infty}$ such that f_n converge to zero pointwise on $[0, 1]$ (as $n \rightarrow \infty$), but f_n do not converge to zero in $L^2([0, 1])$. (2p)

- 4 Formulate and prove Bessel's inequality (any version is fine). (4p)

- 5 Using that $f(x) = e^{-|x|}$ has Fourier transform $\hat{f}(\xi) = 2/(\xi^2 + 1)$, compute for all real values of a the integral

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{(x^2 + 1)^2} dx. \quad (4p)$$