

# Fourier analysis (MMG710/TMA362)

**Time:** 2012-08-22, 8:30–12:30.

**Tools:** Only the attached sheet of formulas. No calculator or handbook is allowed.

**Questions:** Dawan Mustafa, 0703-088304.

**Grades:** Unless otherwise indicated, each problem gives 4 points. For MMG710 grades are G (12-17 points) and VG (18-24 points). For TMA362 grades are 3 (12-14 points), 4 (15-17 points) and 5 (18-24 points).

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- 1 Find the polynomial  $p(x) = ax + b$  that minimizes the integral

$$\int_0^1 |x^2 - p(x)|^2 dx.$$

- 2 Find a function  $u$  such that

$$\int_{-\infty}^{\infty} u(x-y)e^{-|y|} dy = e^{-x^2}.$$

- 3 (a) Expand  $\cos(x)$  as a Fourier sine series on the interval  $0 < x < \pi$ .  
(b) Compute the value of the series

(2p)

$$\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)^2(2n-1)^2}.$$

(2p)

- (c) Solve the inhomogeneous heat equation

$$\begin{aligned} u'_t &= u''_{xx} + \cos(x), & 0 < x < \pi, & \quad t > 0, \\ u(x, 0) &= u(0, t) = u(\pi, t) = 0. \end{aligned}$$

(4p)

- 4 Find a function with Laplace transform

$$\frac{1 + e^{-\pi s}}{(1 - e^{-\pi s})(1 + s^2)}.$$

Sketch the graph of the function.

- 5 Prove the inversion formula for Fourier series. You may assume that the function you are dealing with is continuous.

Good luck!

Hjalmar