Fourier analysis (MMG710/TMA362)

Time: 2013-01-12, 8:30–12:30.

Tools: Only the attached sheet of formulas. No calculator or handbook is allowed.

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Grades: Each problem gives 4 points. For MMG710 grades are G (12-17 points) and VG (18-24). For TMA362 grades are 3 (12-14 points), 4 (15-17) and 5 (18-24).

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1 Expand x as a Fourier sine series on $0 < x < \pi$. Use this result and a theorem on integration of Fourier series to expand x^2 as a Fourier cosine series on $0 < x < \pi$.

2 Let $f(x)$ be given by the Fourier series

$$f(x) = \sum_{n=0}^{\infty} \frac{\cos(nx)}{n^2 + 1}.$$

For which values of A and B is the integral

$$\int_0^{2\pi} |f(x) - A\cos x - B\sin x|^2 \, dx$$

minimal?

3 Using the method of Fourier, find a function $u(x,t)$ on $0 < x < \pi$, $t > 0$ such that

$$\begin{cases} u_{tt} = 4u_{xx}, \\ u(0,t) = u(\pi,t) = 0, \\ u(x,0) = 0, \qquad u_t(x,0) = e^x. \end{cases}$$

4 Using Fourier methods, compute the integral

$$\int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 \cos(3x)\, dx.$$

5 (a) Give an example of a piecewise continuous function on the interval $[-\pi, \pi]$, which is not piecewise C^1.

(b) Give an example of a function on $[-\pi, \pi]$, which is continuous except for jump discontinuities, but is not piecewise continuous.

(In Folland's book, piecewise C^1 functions are called piecewise smooth.)

6 Prove Bessel's inequality (any version is fine).