Fourier analysis (MMG710/TMA362)

Time: 2013-08-21, 8:30–12:30.

Tools: Only the attached sheet of formulas. No calculator or handbook is allowed.

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Grades: Each problem gives 4 points. For MMG710 grades are G (12-17 points) and VG (18-24). For TMA362 grades are 3 (12-14 points), 4 (15-17) and 5 (18-24).

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1 Expand $f(x) = x$ as a Fourier cosine series on the interval $0 < x < \pi$.

2 Find numbers A and B such that

$$\int_0^\infty \left|e^{-x} - Ae^{-2x} - Be^{-3x}\right|^2 dx$$

is minimal.

3 Let

$$f(t) = \begin{cases} e^{-t}, & 1 < t < 2, \\ 0, & \text{else.} \end{cases}$$

Using Laplace transform, solve the initial value problem

$$x' + x = f, \qquad x(0) = 0.$$

4 Solve the inhomogeneous heat equation

$$\begin{aligned} u_t &= u_{xx} + \sin(5x), \\ u(0,t) = 0, \qquad u(\pi,t) &= 1, \qquad u(x,0) = 0, \end{aligned}$$

in the domain $0 < x < \pi$, $t > 0$. You may use without proof the Fourier sine series expansion

$$x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx), \qquad 0 < x < \pi.$$

5 (a) Formulate and prove a theorem on differentiation of Fourier series. (3p)

(b) Is it true or false that

$$1 = 2\sum_{n=1}^{\infty} (-1)^{n+1} \cos(nx), \qquad 0 < x < \pi?$$

If you like, you may use without proof the identity given in problem **4.** (1p)

6 Find all functions $f \in L^1(\mathbb{R})$ such that

$$\int_{-\infty}^{\infty} f(y-x)e^{-x^2}\, dx = e^{-|y|}.$$

Motivate your answer carefully!