

## Additional exercises on Fourier series

1. Find the Fourier series for the following  $2\pi$ -periodic functions, both in sin/cosinus and complex forms, by using elementary algebra instead of integration

$$a) f(x) = \sin^2 x, \quad b) f(x) = (\sin^2 x) \cos x, \quad c) f(x) = \cos^3 x$$

2. (Fourier analysis are very much related to complex analysis. The following exercises are simple examples of such relation.)

a) Find the sum of the following complex Fourier series.

$$\sum_{n=0}^{\infty} r^n e^{in\theta}, \quad 0 \leq r < 1, \quad (1)$$

$$\sum_{n=0}^{\infty} \frac{1}{n} e^{in\theta}, \quad (2)$$

(Hint: Write  $z = re^{i\theta}$  resp.  $z = e^{i\theta}$  and compare with power series expansion of analytic functions.)

b) (More demanding question). Compute the real part of the above expansions to get Fourier sin/cos-series of real-valued functions.