Home work. Part 3[1].

1. (a) Compute the Fourier transforms of the functions

$$f(x) = x^2 e^{-x^2}, \; g(x) = \frac{x}{1 + x^2}$$

(b) Are f and g in the space $L^1(\mathbb{R})$?

2. (a) Find the Laplace transforms of the following functions

$$f(t) = t^3 \sin t, \, g(t) = t \sinh^2 t$$

(Hint: Use "double-angle-formula" for $\sinh t$.)

(b) You might have observed that Laplace transforms of many familiar functions are rational functions. Prove the following general claim: Let $p(t)$ be a polynomial. Then the Laplace transform $F(z) = \mathcal{L}[f](z)$ of of $f(t) = p(t)e^{ct}$ is a rational function of z. (Recall that $F(z)$ is called rational if $F(z) = \frac{P(z)}{Q(z)}$ where P and Q are polynomials)

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[1]The deadline for submitting this home work is Thursday Oct. 13