MATEMATIK
Göteborgs universitet

Datum: 170531
Klockan: 14.00–18.00
Hjälpmedel: Inga
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Jan Stevens

Tentamensskrivning i MMG720, Differentialgeometri

1. Viviani's curve is the intersection of a sphere of radius 2 and a circular cylinder of radius 1 passing through the origin. It can be parametrised as

$$\boldsymbol{\gamma}(t) = (1 + \cos 2t, \sin 2t, 2\sin t)$$

with $-\pi \leq t < \pi$. Compute the curvature and torsion of this curve.

2. Prove the Four Vertex Theorem: every convex simple closed curve in $\mathbb{R}^2$ has at least four vertices.

3. For a surface patch $\boldsymbol{\sigma}\colon \mathbb{R}^2 \to \mathbb{R}^4$ of a surface in 4-space one can define the first fundamental form in the usual way by $E = \boldsymbol{\sigma}_u \cdot \boldsymbol{\sigma}_u$, $F = \boldsymbol{\sigma}_u \cdot \boldsymbol{\sigma}_v$ and $G = \boldsymbol{\sigma}_v \cdot \boldsymbol{\sigma}_v$.
Consider a torus in $\mathbb{R}^4$, parametrised with

$$\boldsymbol{\sigma}(u, v) = (a\cos u, a\sin u, b\cos v, b\sin v)\ ,$$

where a and b are positive constants. Show that this torus is locally isometric to the plane.

4. Compute the principal curvatures of the surface

$$y\cos\frac{z}{a} = x\sin\frac{z}{a}$$

where a is a non-zero constant.
Hint: first parametrise the surface, say with (r, φ) coordinates, so that $z = a\varphi$.

5. a) Show that a geodesic $\boldsymbol{\gamma}$ on a surface S, which is also a line of curvature, is a plane curve.
b) Show that a geodesic with nowhere vanishing curvature, which lies in a plane, is a line of curvature.
c) Give an example of a line of curvature, which lies in a plane and is not a geodesic.

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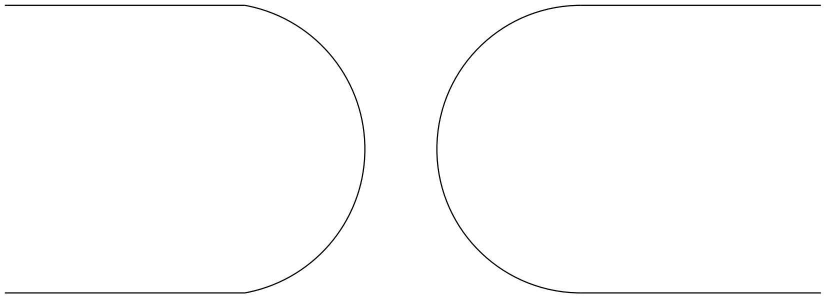
6. Describe (qualitatively) the geodesics on the surface of revolution $M \subset \mathbb{R}^3$, given by

$$\begin{aligned} M = \{(x,y,z) \mid z = 1,\ x^2+y^2 \geq 1\} \\ \cup \{(x,y,z) \mid z = -1,\ x^2+y^2 \geq 1\} \\ \cup \{(x,y,z) \mid |z| \leq 1,\ x^2+y^2 = \varphi(z)\}\ , \end{aligned}$$

where $\varphi(1) = \varphi(-1) = 1$ and $\varphi(z)$ is such that M is a smooth surface, with profile curve looking like



(5p)

Varje uppgift (utom en) ger maximalt 4 poäng. För godkänd skrivning krävs minst 12 poäng. För väl godkänd krävs minst 18 poäng.
Tentan räknas vara färdigrättad onsdagen den 14 juni.
Lycka till! Jan Stevens