HOMEWORK 1

1. Determine the curvature, the torsion and the Frenet trihedron $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ for the curve

$$\gamma(t) = (\ln t, 2t, t^2) \ .$$

Note that the curve is not unit speed.

2. a) Show that the (signed) curvature for a curve in polar coordinates (r, θ) is given by

$$\kappa_s = \frac{r^2 + 2r_\theta^2 - r r_{\theta\theta}}{(r^2 + r_\theta^2)^{\frac{3}{2}}} \ ,$$

where r_θ denotes $\frac{dr}{d\theta}$.
Hint: derive the formulas $x = r(\theta)\cos\theta$, $y = r(\theta)\sin\theta$ with respect to θ.
b) Compute the signed curvature for the *cardioid*

$$r(\theta) = 1 - \sin\theta \ .$$

Sketch the curve with a suitable plotting tool.

3. Let $\gamma : I \to \mathbb{R}^2$ be a smooth planar regular unit-speed curve with positive curvature. Its *evolute* $\varepsilon : I \to \mathbb{R}^2$ is the curve such that the tangent line to ε in the point $\varepsilon(s)$ intersects the curve γ in the point $\gamma(s)$ under a right angle.
It might help to make a sketch.
a) Give a formula for the evolute in parameter form. You may suppose that the curve γ has unit speed.
b) Compute the evolute of the parabola $y = x^2$.
Observe that this curve is not unit speed, but you should not try to reparametrise it. Instead compute the ingredients in your formula directly from the given parametrisation.

Solutions to be handed in at the latest Thursday April 4. js