MATEMATIK
Göteborgs universitet

MMG720 Differentialgeometri vt19

HOMEWORK 3

1. Compute the first fundamental form for the paraboloid

$$\sigma(u,v) = (au\cos v, bu\sin v, u^2)\ .$$

Plot the surface. Write also its equation in Cartesian coordinates.

2. Consider the surface of revolution obtained by rotating the graph of a smooth, positive function $z = f(x)$ around the z-axis. For which functions f is the surface patch

$$\sigma(u,v) = (u\cos v, u\sin v, f(u))$$

conformal?

3. Show that

$$f(x,y,z) = \frac{1}{z^2+1}(x^2-y^2, 2xy, 2z)$$

defines a (smooth) function $f\colon S^2 \to S^2$, where $S^2 = \{p \in \mathbb{R}^3 \mid |p| = 1\}$.
Choose a basis for $T_p(S^2)$ in every $p \in S^2$ (except possible the north and south poles) and determine the matrix for the linear map $D_pf\colon T_p(S^2) \to T_{f(p)}(S^2)$ with respect to this basis.
Hint: you can either use the coordinates (x,y,z) and choose as asked two vectors, spanning the tangent space, in a point and its image point, and compute the matrix by using Df in $\mathbb{R}^3$, or you first write the map in a local patch using a suitable parametrisation of the sphere; in cylindrical coordinates $(x,y,z) = (r\cos\varphi, r\sin\varphi, z)$ on $\mathbb{R}^3$ the sphere is given by $r^2 + z^2 = 1$. You can also use stereographic projection.

Solutions to be handed in Thursday May 9.