MATEMATIK
Göteborgs universitet

Datum: 190605
Klockan: 14.00–18.00
Hjälpmedel: Inga
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Jan Stevens

Tentamensskrivning i MMG720, Differentialgeometri

1. Compute the curvature of the equiangular spiral, given in polar coordinates by the equation $r = ae^{b\vartheta}$, where a and b are positive constants.

2. Prove the Isoperimetric Inequality for simple closed curves; you may assume Wirtinger's Inequality.

3. Define the tangent space at a point P of a smooth surface S and show that it is a two-dimensional vector space.

4. Let $\boldsymbol{\gamma}\colon (\alpha, \beta) \to \mathbb{R}^3$ be a regular unit-speed curve with curvature $\kappa(s) \neq 0$ for all $s \in (\alpha, \beta)$. Let $\{\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}\}$ be its Frenet trihedron. The ruled surface

$$\boldsymbol{\sigma}(s, v) = \boldsymbol{\gamma}(s) + v\boldsymbol{n}(s), \qquad (s, v) \in (\alpha, \beta) \times \mathbb{R}\,,$$

is called the principal normal surface.
a) Determine the points where the parametrisation $\boldsymbol{\sigma}$ is not regular.
b) Compute the Gaussian curvature K. (5p)

5. a) Formulate the Gauss-Bonnet theorem for curvilinear polygons.
b) Verify the Theorem for the curve in the plane consisting of a half circle with radius r and a diameter, by computing both sides of the formula separately.

6. Describe (qualitatively) the geodesics on a torus.

Varje uppgift (utom en) ger maximalt 4 poäng. För godkänd skrivning krävs minst 12 poäng. För väl godkänd krävs minst 18 poäng.
Tentan räknas vara färdigrättad onsdagen den 19 juni.
Lycka till! Jan Stevens