MATEMATIK
Göteborgs universitet

Datum: 090307
Klockan: 8.30–13.30
Hjälpmedel: Inga
Telefon: 076-272 1861
Aron Lagerberg

Tentamensskrivning i MMG720, Differentialgeometri

1. Prove the Four Vertex Theorem: every convex simple closed curve in $\mathbb{R}^2$ has at least four vertices.

2. Let $\boldsymbol{\gamma}(s)$, $s \in [0, l]$, be a positively oriented closed convex plane curve. Let $l(\boldsymbol{\gamma})$ be its length and $\mathcal{A}(\mathrm{int}(\boldsymbol{\gamma}))$ be the area of its interior. The curve

$$\boldsymbol{\delta}(s) = \boldsymbol{\gamma}(s) - r\boldsymbol{n}(s) ,$$

where r is a positive constant, and $\boldsymbol{n}$ is the normal vector, is called a parallel curve to $\boldsymbol{\gamma}$. Show that

a) $l(\boldsymbol{\delta}) = l(\boldsymbol{\gamma}) + 2\pi r$,
b) $\mathcal{A}(\mathrm{int}(\boldsymbol{\delta})) = \mathcal{A}(\mathrm{int}(\boldsymbol{\gamma})) + rl(\boldsymbol{\gamma}) + \pi r^2$,
c) $k_\delta(s) = k_\gamma(s)/(1 + rk_\gamma(s))$, where k_δ and k_γ are the curvatures of the curves $\boldsymbol{\delta}$ and $\boldsymbol{\gamma}$, respectively.

3. Define the tangent space at a point P of a smooth surface S and show that it is a two-dimensional vector space.

4. Show that Scherk's surface

$$z = \ln\left(\frac{\cos y}{\cos x}\right)$$

is a minimal surface, i.e., $H \equiv 0$.

5. Describe the geodesics on a torus.

6. a) Show that a geodesic $\boldsymbol{\gamma}$ on a surface S, which is also a line of curvature, is a plane curve.
b) Show that a geodesic with nowhere vanishing curvature, which lies in a plane, is a line of curvature.
c) Give an example of a line of curvature, which lies in a plane and is not a geodesic.
(5p).

Varje uppgift (utom en) ger maximalt 4 poäng. För godkänd skrivning krävs minst 12 poäng. För väl godkänd krävs minst 18 poäng.
Tentan räknas vara färdigrättad fredagen den 20 mars.
Lycka till! Jan Stevens