MATEMATIK
Göteborgs universitet

Datum: 130314
Klockan: 8.30–12.30
Hjälpmedel: Inga
Telefon: 0703-088304
Jacob Hultgren

Tentamensskrivning i MMG720, Differentialgeometri

1. Compute the signed curvature for the logarithmic spiral $\boldsymbol{\gamma}(t) = (e^{at}\cos t, e^{at}\sin t)$.

2. Prove the Isoperimetric Inequality for simple closed curves; you may assume Wirtinger's Inequality.

3. Let $\boldsymbol{\gamma}(t)$ be a positively oriented regular plane curve and λ a constant. The *parallel curve* $\boldsymbol{\gamma}_\lambda$ of $\boldsymbol{\gamma}$ is defined by

$$\boldsymbol{\gamma}_\lambda(t) = \boldsymbol{\gamma}(t) + \lambda\boldsymbol{n}(t) ,$$

where $\boldsymbol{n}(t)$ is the positive unit normal vector. Show that
a) $\boldsymbol{\gamma}_\lambda$ is a regular curve, if $\lambda k_s < 1$ for all t,
b) the signed curvature of $\boldsymbol{\gamma}_\lambda$ is $k_s/(1-\lambda k_s)$; here k_s is the signed curvature of $\boldsymbol{\gamma}$.

4. Find the principal curvatures and the principal vectors associated to each principal curvature for the Enneper surface, parametrised by

$$\boldsymbol{\sigma}(u,v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + u^2v, u^2 - v^2) .$$

5. Let $\boldsymbol{\sigma}$ be the ruled surface

$$\boldsymbol{\sigma}(u,v) = \boldsymbol{\gamma}(u) + v\boldsymbol{\delta}(u) ,$$

where $\boldsymbol{\gamma}$ is a unit-speed curve in $\mathbb{R}^3$ and $\boldsymbol{\delta}(u)$ a unit vector for all u. Prove that the map $\boldsymbol{\sigma}\colon \mathbb{R}^2 \supset U \to \boldsymbol{\sigma}(U) \subset \mathbb{R}^3$ is an isometry if and only if $\boldsymbol{\delta}(u)$ is constant and $\boldsymbol{\gamma}$ lies in a plane perpendicular to $\boldsymbol{\delta}$. What kind of surface is $\boldsymbol{\sigma}$ in this case?

6. Describe the geodesics on a spheroid, the surface of revolution obtained by rotating an ellipse around one of its axes. (5p).

Every exercise (except one) gives at most 4 points. For the grade Pass (G) 12 points are needed, for Pass with Distinction (VG) 18 points.
The results will be reported before April 8th in LADOK.

Lycka till! Jan Stevens