MATEMATIK
Göteborgs universitet

Datum: 150603
Klockan: 14.00–18.00
Hjälpmedel: Inga
Telefon: 0709322268
Jan Stevens

Tentamensskrivning i MMG720, Differentialgeometri

1. Compute the curvature and torsion for the curve

$$\gamma(t) = (3t - t^3, 3t^2, 3t + t^3) \ .$$

2. Prove the Four Vertex Theorem: every convex simple closed curve in $\mathbb{R}^2$ has at least four vertices.

3. Define the tangent space at a point P of a smooth surface S and show that it is a two-dimensional vector space.

4. Consider the Möbius band with parametrisation

$$\sigma(t,\vartheta) = ((1 - t\sin\tfrac{\vartheta}{2})\cos\vartheta, (1 - t\sin\tfrac{\vartheta}{2})\sin\vartheta, t\cos\tfrac{\vartheta}{2}) \ ,$$

where $-\frac{1}{2} < t < \frac{1}{2}$ and $0 < \vartheta < 2\pi$. Compute the Gaussian curvature K on the circle given by $t = 0$.

5. Show that Scherk's surface

$$z = \ln\left(\frac{\cos y}{\cos x}\right)$$

is a minimal surface, i.e., $H \equiv 0$.

6. Describe the geodesics on the circular cone

$$S = \{(x,y,z) \in \mathbb{R}^3 \mid z^2 = a^2(x^2+y^2),\ z > 0 \} \ ,$$

where $a \in \mathbb{R}_+$. For which values of a do there exist geodesics that intersect themselves?
(5p)

Varje uppgift (utom en) ger maximalt 4 poäng. För godkänd skrivning krävs minst 12 poäng. För väl godkänd krävs minst 18 poäng.
Tentan räknas vara färdigrättad fredagen den 12 juni.
Lycka till! Jan Stevens