

# Lösningar tenta MMGF20

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1. a) Riktningensderivatan ges av  $\nabla f(1, -2) \cdot v$ ,  
där  $v$  är normeringen av  $(3, -1)$ .

$$\nabla f = (e^{xy} + xye^{xy} + 2xy, x^2 e^{xy} + x^2)$$

$$\nabla f(1, -2) = (e^{-2} - 2e^{-2} + 4, e^{-2} + 1) = (4 - e^{-2}, 1 + e^{-2})$$

$$v = \frac{1}{\sqrt{3^2 + 1^2}} (3, -1) = \frac{1}{\sqrt{10}} (3, -1)$$

$$\begin{aligned} \nabla f(1, -2) \cdot v &= \left( (4 - e^{-2}) \cdot 3 + (1 + e^{-2}) \cdot (-1) \right) \frac{1}{\sqrt{10}} = \\ &= \frac{1}{\sqrt{10}} (11 - 4e^{-2}) \end{aligned}$$

b.) I riktningen  $\nabla f(1, -2) = (4 - e^{-2}, 1 + e^{-2})$ .

2) Vi letar först efter stationära punkter i områdets inre:

$$\begin{cases} f'_x = 3x^2 - 2y^2 + 1 = 0 \\ f'_y = 4xy = 0 \end{cases}$$

Slutligen (2) ger att  $x = 0$  eller  $y = 0$ .

$$x = 0 \text{ ger } -2y^2 + 1 = 0 \Leftrightarrow 2y^2 = 1$$

$$\Leftrightarrow y = \pm \frac{1}{\sqrt{2}}, \text{ dvs } \boxed{(0, \pm \frac{1}{\sqrt{2}})}$$

$$y = 0 \text{ ger } 3x^2 + 1 = 0 \Leftrightarrow x^2 = -\frac{1}{3},$$

som saknar lösning.

Vi tittar nu på randvärdena:

$$\underline{x=1}: f(1, y) = 1 - 2y^2 + 1 + 1 = 3 - 2y^2 = g_1(y)$$

$$g_1'(y) = -4y = 0 \Rightarrow y = 0, \text{ dvs } \boxed{(1, 0)}$$

$$\underline{x=-1}: f(-1, y) = -1 + 2y^2 - 1 + 1 = 2y^2 - 1 = g_2(y)$$

$$g_2'(y) = 4y = 0 \Rightarrow y = 0, \text{ dvs } \boxed{(-1, 0)}$$

$$\underline{y=\pm 1}: f(x, \pm 1) = x^3 - 2x + x + 1 = x^3 - x + 1 = g_3(x)$$

$$g_3'(x) = 3x^2 - 1 = 0 \Leftrightarrow$$

$$x^2 = \frac{1}{3} \Leftrightarrow x = \pm \frac{1}{\sqrt{3}}$$

$$\boxed{\begin{matrix} (\frac{1}{\sqrt{3}}, \pm 1) \\ (-\frac{1}{\sqrt{3}}, \pm 1) \end{matrix}}$$

Vi måste också titta på värdena

i hörnen:  $\boxed{(1, \pm 1) \quad (-1, \pm 1)}$ .

Nu jämför vi alla värden:

$$f(0, \pm \frac{1}{\sqrt{2}}) = 1 \quad f(1, 0) = 1 + 1 + 1 = 3$$

$$f(-1, 0) = -1 - 1 + 1 = -1 \quad f(1, \pm 1) = 1 - 2 + 1 + 1 = 1$$

$$f(-1, \pm 1) = -1 + 2 - 1 + 1 = 1$$

$$f\left(\frac{1}{\sqrt{3}}, \pm 1\right) = \frac{1}{3\sqrt{3}} - \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} + 1 = \frac{1 - 6 + 3}{3\sqrt{3}} + 1 = 1 - \frac{2}{3\sqrt{3}}$$

som ligger mellan 0 och 1.

$$f\left(-\frac{1}{\sqrt{3}}, \pm 1\right) = -\frac{1}{3\sqrt{3}} + \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} + 1 = 1 + \frac{2}{3\sqrt{3}}$$

som ligger mellan 1 och 2.

$$\text{Max: } \underline{\underline{f(1, 0) = 3}}$$

$$\text{Min: } \underline{\underline{f(-1, 0) = -1}}$$

③ Svaret på första frågan är ja om  $f'_y(1, \pi) \neq 0$ , enligt implicita funktions-satsen.

$$f'_y = x \cos(y) + x e^{xy}$$

$$f'_y(1, \pi) = \cos(\pi) + e^\pi = e^\pi - 1 \neq 0.$$

Nu deriverar vi  $f(x, y(x))$  implicit m.a.p.  $x$ :

$$\sin(y(x)) + x \cos(y(x)) \cdot y'(x) + e^{xy(x)} \cdot (y(x) + x y'(x)) = 0.$$

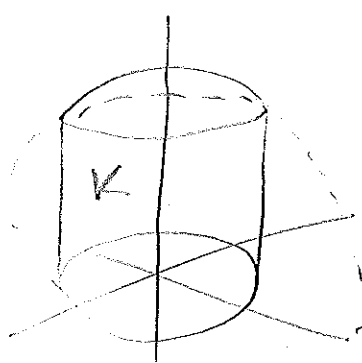
Sätt in  $(x, y) = (1, \pi)$ :

$$\underbrace{\sin(\pi)}_{=0} + \underbrace{\cos(\pi)}_{=-1} \cdot y'(1) + e^\pi (\pi + y'(1)) = 0$$

$$y'(1) (e^\pi - 1) = -\pi e^\pi$$

$$y'(1) = -\frac{\pi e^\pi}{e^\pi - 1} = \frac{-\pi}{1 - e^{-\pi}}$$

④ Av symmetrihal är  $x_T = y_T = 0$ .



$$m = \iiint_K dx dy dz =$$

$$= \iint_{x^2 + y^2 \leq 1} \left( \int_0^{\sqrt{4-x^2-y^2}} dz \right) dx dy =$$

$$\left\{ \begin{array}{l} \text{polära} \\ \text{koord.} \end{array} \right\} = \int_0^{2\pi} \int_0^1 \sqrt{4-r^2} r dr d\theta = 2\pi \left[ -\frac{(4-r^2)^{3/2}}{3/2 \cdot 2} \right]_0^1 =$$

$$= -\frac{2\pi}{3} \left( 3^{3/2} - 4^{3/2} \right) = -\frac{2\pi}{3} (3\sqrt{3} - 8) = \frac{2\pi}{3} (8 - 3\sqrt{3})$$

$$\begin{aligned}
\iiint_K z \, dx \, dy \, dz &= \iint_{x^2+y^2 \leq 1} \left( \int_0^{\sqrt{4-x^2-y^2}} z \, dz \right) dx \, dy = \\
&= \iint_{x^2+y^2 \leq 1} \left[ \frac{z^2}{2} \right]_0^{\sqrt{4-x^2-y^2}} dx \, dy = \int_0^{2\pi} \int_0^1 \frac{4-r^2}{2} r \, dr \, d\theta = \\
&= \frac{2\pi}{2} \int_0^1 4r - r^3 \, dr = \pi \left[ 2r^2 - \frac{r^4}{4} \right]_0^1 = \\
&= \pi \left( 2 - \frac{1}{4} \right) = \frac{7\pi}{4}.
\end{aligned}$$

$$\int_a^b z_T = \frac{\frac{7\pi}{4}}{\frac{2\pi}{3}(8-3\sqrt{3})} = \frac{7}{4} \cdot \frac{3}{2} \cdot \frac{1}{8-3\sqrt{3}} = \frac{21}{8(8-3\sqrt{3})}$$

5)  $\mathbf{r}'(t) = (-1, 1, 2t)$ ,  $\int_a^b \mathbf{F} \cdot d\mathbf{r} =$

$$\begin{aligned}
&= \int_0^1 (1-t) \cdot t^2 \cdot (-1) + t e^{1-t} \cdot (-1) + e^{t^2} \cdot 2t \, dt = \\
&= \int_0^1 t^3 - t^2 - t e^{1-t} + 2t e^{t^2} \, dt = \\
&= \left[ \frac{t^4}{4} - \frac{t^3}{3} + e^{t^2} \right]_0^1 - \int_0^1 t e^{-t} \, dt = \\
&= \frac{1}{4} - \frac{1}{3} + e^1 - e^0 - \left[ -t e^{-t} \right]_0^1 + \int_0^1 -e^{-t} \, dt = \\
&= e + \frac{3-4}{12} - 1 + e^{-1} - e^0 + \left[ e^{-t} \right]_0^1 = \\
&= e - 2e^{-1} - \frac{1}{12} - 3 = e - 2e^{-1} - \frac{37}{12}
\end{aligned}$$

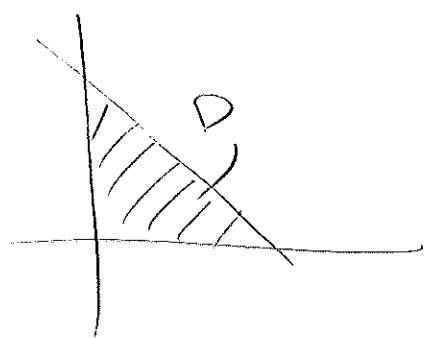
← partial-integrations

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$$\textcircled{6.} \int_{\gamma} \underbrace{(\ln(1+x^2) + xy^2)}_P dx + \underbrace{(e^{\sin(y)} + xy^2)}_Q dy = \begin{cases} \text{Green} \\ \text{Satz} \end{cases}$$

$$= \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \iint_D (y^2 - 2xy) dx dy = (*)$$

dar  $D = \{(x,y) : x \geq 0, y \geq 0, x+y \leq 1\}$ .



$$(*) = \int_0^1 \int_0^{1-y} (y^2 - 2xy) dx dy =$$

$$= \int_0^1 [xy^2 - x^2y]_0^{1-y} dy =$$

$$= \int_0^1 (1-y)y^2 - (1-y)^2 y dy = \int_0^1 y^2 - y^3 - (1-2y+y^2)y dy$$

$$= \int_0^1 -2y^3 + 3y^2 - y dy = \left[ -\frac{2y^4}{4} + y^3 - \frac{y^2}{2} \right]_0^1 =$$

$$= -\frac{1}{2} + 1 - \frac{1}{2} = \underline{\underline{0}}$$

7.) Se bohun

8.) a.) Se bohun

b.) Se bohun

c.) VS (a,b,c).