Approximate amenability of tensor products of Banach algebras

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In memoriam

Charles John Read - mathematician, gentelman and friend

$$A = \text{a Banach algebra}$$

$$X = \text{a Banach } A\text{-bimodule}$$

derivation $D : A \longrightarrow X$ is **inner** if $\exists x \in X$ such that

$$D(a) = a \cdot x - x \cdot a, \quad (a \in A), \qquad D = \mathrm{ad}_x$$

All derivations in this talk are **continuous**

D is **approximately inner** if $\exists (x_i) \subset X$, such that

$$\begin{aligned} D(a) &= \lim_i a.x_i - x_i.a \qquad (a \in A), \\ &= \lim_i \mathrm{ad}_{x_i}(a). \end{aligned}$$

A is **approximately amenable** if every $D : A \longrightarrow X^*$ is approximately inner for all X.

Boundedly approximately amenable:

$D = \lim \mathrm{ad}_{x_i^*}$ (SO), for some operator-norm bounded net $(\mathrm{ad}_{x_i^*})$.

Approximately contractible

$D : A \longrightarrow X, \ldots$

Boundedly approximating contractible ...

Weak* approximate amenability

Approximation is in weak*-topology.

Equivalences

(**R. J. Loy, Y. Zhang, F. Gh.**, 2004)

Approx. amen. $\Leftrightarrow$ Approx. contract. $\Leftrightarrow$ Weak*-approx. amen.

Does equivalences hold for “bounded” versions?

(**C. J. Read, F. Gh.**, 2010) There exists a boundedly approximately amenable Banach algebra A, which is not boundedly approximately contractile .

For this A the Banach algebra $A \oplus A^{op}$ is not approximately amenable.

What about bdd. approx. amen. compared to approx. amen?

(**C. J. Read, F. Gh.**, 2013). There exists an approximately amenable Banach algebra which is not boundedly approximately amenable.