Approximate amenability of tensor products of Banach algebras

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In memoriam

Charles John Read - mathematician, gentleman and friend
\[ A = \text{a Banach algebra} \]

\[ X = \text{a Banach } A\text{-bimodule} \]

derivation \( D : A \rightarrow X \) is **inner** if \( \exists x \in X \) such that

\[ D(a) = a \cdot x - x \cdot a, \quad (a \in A), \quad D = \text{ad}_x \]
All derivations in this talk are **continuous**

* $D$ is approximately inner* if $\exists (x_i) \subset X$, such that

\[
D(a) = \lim_i a.x_i - x_i.a \quad (a \in A),
\]

\[
= \lim_i \text{ad}_{x_i}(a).
\]
A is **approximately amenable** if every $D : A \rightarrow X^*$ is approximately inner for all $X$.

**Boundedly approximately amenable:**

$$D = \lim \text{ad}_{x_i^*}(SO),$$ for some operator-norm bounded net $(\text{ad}_{x_i^*})$. 
Approximately contractible

\[ D : A \rightarrow X, \ldots \]

Boundedly approximating contractible ...
Approximation is in weak*-topology.
Approx. amen. $\Leftrightarrow$ Approx. contract. $\Leftrightarrow$ Weak*-approx. amen.

(R. J. Loy, Y. Zhang, F. Gh., 2004)
Does equivalences hold for “bounded” versions?

(C. J. Read, F. Gh., 2010) There exists a boundedly approximately amenable Banach algebra $A$, which is not boundedly approximately contractile.

For this $A$ the Banach algebra $A \oplus A^{op}$ is not approximately amenable.
What about bdd. approx. amen. compared to approx. amen?

(C. J. Read, F. Gh., 2013). There exists an approximately amenable Banach algebra which is not boundedly approximately amenable.