Lösn. Naturvetarmatematik A1 MMGK11, del 2 — 120316

1) a) $\int x\cos x^2\,dx = [t=x^2] = \int (\cos t)\frac{1}{2}\,dt = \frac{1}{2}\sin t + C = \frac{1}{2}\sin x^2 + C$

b) $\int_0^\pi x\cos x\,dx = [PI] = [x\sin x]_0^\pi - \int_0^\pi 1\cdot\sin x\,dx = 0 - [-\cos x]_0^\pi = -1-1 = -2$

c) $\int \cos^2 x\,dx = \frac{1}{2}\int 1+\cos(2x)\,dx = \frac{1}{2}\left(x + \frac{\sin(2x)}{2}\right) + C$

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2) $y' + y = 1$ linjär ⇒ IF: $e^{\int 1\,dx} = e^x$ ⇒ $\frac{d}{dx}(e^x y) = e^x$ ⇒ $e^x y = \int \frac{d}{dx}(e^x y)\,dx = \int e^x\,dx = e^x + C$

∴ $y = 1 + Ce^{-x}$ och $2 = y(0) = 1 + C\cdot 1$ ⇒ $C = 1$ ⇒ Sökt lösn. $y(x) = 1 + e^{-x}$

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3) $y = y_p + y_h$. För y_h: kar. ekv. $0 = r^2 - 4r + 3 = (r-1)(r-3)$ ⇒ $y_h = C_1e^x + C_2e^{3x}$. För y_p:
Ansats $y_p = x^m(Ce^x) = (m=1) = Cxe^x$. Insättning i ekv ⇒ $2e^x =$
$= y_p'' - 4y_p' + 3y_p = (2C + Cx)e^x - 4(C + Cx)e^x + 3Cxe^x = -2Ce^x$ ⇒ $C = -1$ ⇒ $y_p = -xe^x$

∴ $0 = y(0) = C_1 + C_2$ och $0 = y'(0) = -1 + C_1 + 3C_2$ ∴ $C_1 = -1/2$, $C_2 = 1/2$

∴ Sökt lösn. $= y = -xe^x - \frac{1}{2}e^x + \frac{1}{2}e^{3x} = -(x + \frac{1}{2})e^x + \frac{1}{2}e^{3x}$

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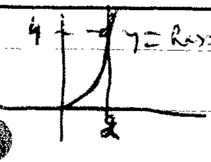
4) 

a) $\sum_i \pi f(x_i)^2\,dx \to V = \int_0^2 \pi f(x)^2\,dx = \pi\int_0^2 x^4\,dx = \pi\left[\frac{x^5}{5}\right]_0^2 = \frac{32\pi}{5}$

b) $\sum_i 2\pi x_i f(x_i)\,dx_i \to V = \int_0^2 2\pi x f(x)\,dx = 2\pi\int_0^2 x^3\,dx = 2\pi\left[\frac{x^4}{4}\right]_0^2 = 8\pi$

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5) $\arctan t = t - \frac{t^3}{3} + \frac{t^5}{5} + \dots$, $\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} + \dots$ ∴ $\lim_{x\to 0}\frac{3\sin x - \sin(3x)}{3\arctan x - \arctan(3x)} =$

$$= \lim_{x\to 0}\frac{3\left(x - \frac{x^3}{3!} + O(x^5)\right) - \left(3x - \frac{(3x)^3}{3!} + O(x^5)\right)}{3\left(x - \frac{x^3}{3} + O(x^5)\right) - \left(3x - \frac{(3x)^3}{3} + O(x^5)\right)} = \lim_{x\to 0}\frac{x^3\left(\left(\frac{3^2}{2!} - \frac{1}{2!}\right) + O(x^2)\right)}{x^3\left((3^2-1) + O(x^2)\right)} = \frac{\frac{9-1}{2} + 0}{9 - 1 + 0} = \frac{4}{8} = \frac{1}{2}$$

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6) $I = \int e^{ax}\sin bx\,dx = [PI] = \frac{e^{ax}}{a}\sin bx - \int \frac{e^{ax}}{a}b\cos bx\,dx = [PI] = \frac{e^{ax}\sin bx}{a}$

$- \frac{b}{a}\left(\frac{e^{ax}}{a}\cos bx - \int\frac{e^{ax}}{a}(-b\sin bx)\,dx\right) = \frac{e^{ax}\sin bx}{a} - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}I$

∴ $\frac{a^2+b^2}{a^2}I = \left(1 + \frac{b^2}{a^2}\right)I = e^{ax}\left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right) + C$

∴ $I = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$

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7) a) $y(t)$ = mängden jäst i behållaren vid tiden t. Tillväxthastighet ~ mängden jäst ⇒ Tillväxthast. $= ky(t)$, $k = 1/5$ ∴ $\frac{dy}{dt} = ky - a$ ⇔ $y' - ky = -a$

linjär, IF $e^{\int -k\,dt} = e^{-kt}$ ∴ $\frac{d}{dt}(e^{-kt}y) = -ae^{-kt}$ ⇒ $e^{-kt}y = \int\frac{d}{dt}(e^{-kt}y)\,dt =$

$= \int -ae^{-kt}\,dt = -a\frac{e^{-kt}}{-k} + C$ ⇒ $y = \frac{a}{k} + Ce^{kt}$, $y(0) = y_0$ ⇒ $C = y_0 - \frac{a}{k}$

∴ $y(t) = \frac{a}{k} + \left(y_0 - \frac{a}{k}\right)e^{kt}$

b) $y_0 - \frac{a}{k} = 0$ ⇒ $a = ky_0 = y_0/5$

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8) Se kurslitteraturen.