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1 a) $\int_0^1 x^3(1+x)dx = \int_0^1 x^3+x^4 dx = \int_0^1 x^3 dx + \int_0^1 x^4 dx = \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^5}{5}\right]_0^1 =$

b) $\int 3\sin^2 x \cos x dx = \begin{bmatrix} t=\sin x \\ \frac{dt}{dx} = \cos x dx \end{bmatrix} = \int 3t^2 dt = t^3 + C = \sin^3 x + C$

c) $\int \cos^3 x dx = \int \cos^2 x \cos x dx = \int (1-\sin^2 x)\cos x dx = \begin{bmatrix} t=\sin x \\ \frac{dt}{dx} = \cos x dx \end{bmatrix} = \int 1-t^2 dt =$
$= t - \frac{t^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$

2) $y'+2xy = 2x$ IF: $e^{\int 2x dx} = e^{x^2} \Rightarrow \frac{d}{dx}(e^{x^2}y) = 2xe^{x^2} \Rightarrow e^{x^2}y = \int \frac{d}{dx}(e^{x^2}y)dx$
$= \int 2xe^{x^2}dx = e^{x^2} + C \Rightarrow y = 1 + Ce^{-x^2}$, $2 = y(0) = 1 + Ce^0 \Rightarrow C = 1 \Rightarrow y = 1 + e^{-x^2}$

3) $y = y_p + y_h$. kar. ekv: $r^2 - 4r + 3 = 0 \Rightarrow (r-1)(r-3) \Rightarrow y = C_1e^x + C_2e^{3x}$
Ansatz $y_p = x^m Ce^{2x} = (m=0) = Ce^{2x}$, $y_p' = 2Ce^{2x}$, $y_p'' = 4Ce^{2x}$ Insättning $\Rightarrow$
$4Ce^{2x} - 4 \cdot 2Ce^{2x} + 3Ce^{2x} = e^{2x} \Leftrightarrow 4C - 8C + 3C = 1 \Rightarrow C = -1 \therefore y_p = -1e^{2x} = -e^{2x}$

4) a) $dist = |(1,1)-(4,5)| = |(-3,-4)| = \sqrt{(-3)^2+(-4)^2} = \sqrt{25} = 5$
b) $(x(t),y(t))$ $\gamma(t) = (1,1) + tv$ $v = \vec{AB} = (4,5)-(1,1) = (3,4)$ $A=(1,1)$, $B=(4,5)$
$dist = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}dt = \int_0^1 \sqrt{3^2+4^2}dt = \int_0^1 5dt = 5\int_0^1 dt = 5 \cdot 1 = 5$

5) ~~Maximera~~ Nämnaren $x\sin x = x(x + \mathcal{O}(x^3)) = x^2 + \mathcal{O}(x^4)$ Så i täljaren
$\cos x - 1 = 1 - \frac{x^2}{2} + \mathcal{O}(x^4) - 1 = -\frac{x^2}{2} + \mathcal{O}(x^4) \therefore \lim_{x\to 0} \frac{\cos x - 1}{x\sin x} = \lim_{x\to 0} \frac{-\frac{x^2}{2} + \mathcal{O}(x^4)}{x^2 + \mathcal{O}(x^4)}$
$= \lim_{x\to 0} \frac{\cancel{x^2}(-\frac{1}{2} + \mathcal{O}(x^2))}{\cancel{x^2}(1+\mathcal{O}(x^2))} = \lim_{x\to 0} \frac{-\frac{1}{2} + \mathcal{O}(x^2)}{1 + \mathcal{O}(x^2)} = \frac{-\frac{1}{2}+0}{1+0} = -\frac{1}{2}$

6) $I = \int e^{ax}\sin bx dx = \{PI\} = \frac{e^{ax}}{a}\sin bx - \frac{b}{a}\int e^{ax}\cos bx dx = \{PI\} = \frac{e^{ax}\sin bx}{a}$
$- \frac{b}{a}\left(\frac{e^{ax}}{a}\cos bx + \frac{b}{a}\int e^{ax}\sin bx dx\right) = \left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right)e^{ax} - \left(\frac{b}{a}\right)^2 I \therefore$
$\left(1+\left(\frac{b}{a}\right)^2\right)I = \left(\frac{\sin bx}{a} - \frac{b}{a^2}\cos bx\right)e^{ax} + C \Rightarrow I = \frac{1}{a^2+b^2} \cdot a^2\left(\frac{a\sin bx - b\cos bx}{a^2}\right)e^{ax} + C$
$= \frac{1}{a^2+b^2}(a\sin bx - b\cos bx)e^{ax} + C$

7) Derivering $\Rightarrow y'(x) = xy(x) \Rightarrow y' - xy = 0$ IF: $e^{\int -x dx} = e^{-x^2/2} \Rightarrow$
$\frac{d}{dx}(e^{-x^2/2}y) = e^{-x^2/2} \cdot 0 = 0 \Rightarrow e^{-x^2/2}y = \int \frac{d}{dx}(e^{-x^2/2}y)dx = \int 0 dx = C \Rightarrow$
$y = Ce^{x^2/2}$ Sätt $x=0$ i ekvationen $\Rightarrow y(0) = 1 + \int_0^0 t\,y(t)dt = 1 + 0 = 1 \therefore 1 = y(0) =$
$= Ce^0 = C \therefore$ Sökt lösning $y(x) = e^{x^2/2}$