Lösningar Naturvetarmatematik A1, del 2 MMGK11, 130830

1. a) $\int_0^{\pi/2}\cos(2x)\,dx = \left[\frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left(\sin\frac{\pi}{4} - \sin 0\right) = \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{4}$ b) $\int x^2(x^2+1)\,dx = \int x^4 + x^2\,dx$ $= x^5/5 + x^3/3 + C$ c) $\int e^{\sqrt{x}}dx = \begin{bmatrix} t=\sqrt{x} \\ x=t^2,\ dx = 2t\,dt\end{bmatrix} = \int e^t\, 2t\,dt = 2\int te^t\,dt = [PI] = 2\left(te^t - \int e^t dt\right)$ $= 2(te^t - e^t + C) = 2(t-1)e^t + C = 2(\sqrt{x}-1)e^{\sqrt{x}} + C$

2) $y' - y = 0$ IF: $e^{\int -1 dx} = e^{-x} \Rightarrow \frac{d}{dx}(e^{-x}y) = 0e^{-x} = 0 \Rightarrow e^{-x}y = C \Rightarrow y = Ce^{x}$, $1 =$ $= y(0) = Ce^0 = C \Rightarrow C = 1 \therefore y = e^{x}$

3) $y = y_p + y_h$: Kar. ekv. $0 = r^2 + 3r - 4 = (r-1)(r+4) \Rightarrow y_h = C_1e^x + C_2e^{-4x}$
Ansatz $y_p = Cx^n e^{2x} = (n=0) = Ce^{2x} \Rightarrow y_p' = 2Ce^{2x}$, $y_p'' = 4Ce^{2x}$ Insättning $\Rightarrow$
$6 = 6C \Rightarrow C = 1 \Rightarrow y_p = e^{2x} \therefore y = e^{2x} + C_1e^x + C_2e^{-4x}$, $y' = 2e^{2x} + C_1e^x - 4C_2e^{-4x}$
$y(0) = y'(0) = 1 \Rightarrow \begin{cases} 1 = 1 + C_1 + C_2 \\ 1 = 2 + C_1 - 4C_2 \end{cases} \Rightarrow C_1 = -\frac{1}{5},\ C_2 = \frac{1}{5} \Rightarrow y = e^{2x} - \frac{1}{5}e^x + \frac{1}{5}e^{-4x}$

4) $L = \int_0^1 \sqrt{1 + (f'(x))^2}\,dx = \int_0^1 \sqrt{1 + (x^{1/2})^2}\,dx = \int_0^1 \sqrt{1+x}\,dx = \left[\frac{2}{3}(1+x)^{3/2}\right]_0^1 = \frac{2}{3}(2^{3/2} - 1)$

5) a) $\lim_{x\to 0} 2\cdot\frac{\sin(2x)}{2x} = 2\lim_{x\to 0}\frac{\sin 2x}{2x} = 2\lim_{t\to 0}\frac{\sin t}{t} = 2\cdot 1 = 2$ b) $\lim_{x\to\infty}\frac{x^3(1 - \frac{1}{x} + \frac{1}{x^3})}{x^3(1 + \frac{1}{x^2})} = \frac{1+0}{1+0}$ $= 1$ c) $\lim_{x\to 0}\frac{\arctan x - x}{\ln(1+x) - x + \cos x - 1} = \lim_{x\to 0}\frac{-\frac{x^3}{3} + O(x^5)}{-\frac{x^2}{2} + O(x^3) - \frac{x^2}{2!} + O(x^4)} = \lim_{x\to 0}\frac{x^2\left(-\frac{x}{3} + O(x^3)\right)}{x^2(-1 + O(x))} = \frac{0+0}{-1+0} = 0$

6) $y'(t) =$ förändringen i jord med tiden $t = ky(t) - a$, $k = 4/10 \Rightarrow y' - ky = -a$
IF: $e^{\int -k\,dt} = e^{-kt} \therefore e^{-kt}y = \int \frac{d}{dt}(e^{-kt}y)\,dt = \int -a e^{-kt}\,dt = \frac{a}{k}e^{-kt} + C \Rightarrow$
$y = \frac{a}{k} + Ce^{kt}$, $y_0 = y(0) = \frac{a}{k} + C \Rightarrow C = y_0 - \frac{a}{k} \therefore y = \frac{a}{k} + (y_0 - \frac{a}{k})e^{kt}$
Välj a så $y_0 - \frac{a}{k} = 0 \Rightarrow a = ky_0 = \frac{4}{10}y_0$

7) $I \equiv \int\sqrt{x^2+1}\,dx = \int\frac{x^2+1}{\sqrt{x^2+1}}dx = \int\frac{x^2}{\sqrt{x^2+1}}dx + \int\frac{1}{\sqrt{x^2+1}}dx = I_1 + I_2$
$I_2 = \ln|x + \sqrt{x^2+1}| + C = \ln(x + \sqrt{x^2+1}) + C$
$I_1 = \int x\frac{x}{\sqrt{x^2+1}}dx = [PI] = x(x^2+1)^{1/2} - \int 1\cdot\sqrt{x^2+1}\,dx + C = x(x^2+1)^{1/2} - I + C$

$\therefore 2I = (\text{bootstrapping}) = x\sqrt{x^2+1} + \ln(x + \sqrt{x^2+1}) + C \Rightarrow$

$\int\sqrt{x^2+1}\,dx = I = \frac{x\sqrt{x^2+1}}{2} + \frac{1}{2}\ln(x + \sqrt{x^2+1}) + C$

8) Se kursboken.