Naturvetenskaplig matematik A1, MMGK11, del 2 — 140320

1a) $\int_0^{\pi/10} \sin 5x\,dx = \left[\frac{-\cos(5x)}{5}\right]_0^{\pi/10} = -\frac{1}{5}\left(\cos\frac{\pi}{2} - \cos 0\right) = -\frac{1}{5}(0-1) = \frac{1}{5}$

b) $\int \tan x\,dx = \int \frac{1}{\cos x}\sin x\,dx = \left[\begin{matrix} t=\cos x \\ \frac{dt}{dx} = -\sin x\end{matrix}\right] = -\int \frac{1}{t}\,dt = -\ln|t| + C = -\ln|\cos x| + C$

c) $\int \cos\sqrt{x}\,dx = \left[\begin{matrix} t=\sqrt{x} \\ x=t^2 \\ \frac{dx}{dt} = 2t\end{matrix}\right] = 2\int t\cos t\,dt = [PI] = 2\left(t\sin t - \int 1\cdot\sin t\,dt\right) = 2t\sin t - 2(-\cos t) + C = 2t\sin t + 2\cos t + C$
$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

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2) Linjär: IF: $e^{\int 1\,dx} = e^x \Rightarrow \frac{d}{dx}(e^x y) = xe^x \Rightarrow e^x y = \int \frac{d}{dx}(e^x y)\,dx = \int xe^x\,dx = [PI]$
$= xe^x - \int 1 e^x\,dx = xe^x - e^x + C \Rightarrow y = (x-1) + \frac{C}{e^x} = x-1+Ce^{-x}$; $1 = y(0) = (0-1) + Ce^0 \Rightarrow C = 2 \Rightarrow y = x-1+2e^{-x}$

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3) $y = y_p + y_h$: y_h: kar. ekv. $0 = r^2 + r - 2 = (r-1)(r+2) \Rightarrow y_h = C_1e^x + C_2e^{-2x}$; y_p: Ansats
$y_p = x^0(Ax+B) = (m=0) = Ax+B \Rightarrow y_p' = A,\ y_p'' = 0$. Insättning av $y_p \Rightarrow x = y_p'' + y_p' - 2y_p =$
$= 0 + A - 2(Ax+B) = -2Ax + A - 2B \Rightarrow \begin{cases} -2A = 1 \\ A - 2B = 0\end{cases} \Rightarrow \begin{cases} A = -\frac{1}{2} \\ B = -\frac{1}{4}\end{cases} \Rightarrow y_p = -\frac{1}{2}x - \frac{1}{4} \Rightarrow y = -\frac{1}{2}x - \frac{1}{4} + C_1e^x + C_2e^{-2x}$

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4) $\lim_{x\to 0} \frac{\arctan x - \ln(1+x)}{\cos x - e^{x^2}} = \lim_{x\to 0} \frac{x + O(x^3) - \left(x - \frac{x^2}{2} + O(x^3)\right)}{1 - \frac{x^2}{2!} + O(x^4) - (1 + x^2 + O(x^4))} = \lim_{x\to 0} \frac{\frac{x^2}{2} + O(x^3)}{-\frac{3}{2}x^2 + O(x^4)} = \lim_{x\to 0} \frac{x^2\left(\frac{1}{2} + O(x)\right)}{x^2\left(-\frac{3}{2} + O(x^2)\right)}$
$= \lim_{x\to 0} \frac{\frac{1}{2} + O(x)}{-\frac{3}{2} + O(x^2)} = \frac{\lim_{x\to 0} \frac{1}{2} + O(x)}{\lim_{x\to 0} -\frac{3}{2} + O(x^2)} = \frac{\frac{1}{2} + 0}{-\frac{3}{2} + 0} = -\frac{1}{3}$

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5) $\int \frac{3x+1}{x^2+2x+2}\,dx = \int \frac{3x+1}{(x+1)^2+1}\,dx = \left[\begin{matrix} t = x+1 \\ \frac{dt}{dx} = 1\end{matrix}\right] = \int \frac{3(t-1)+1}{t^2+1}\,dt = \frac{3}{2}\int \frac{2t}{t^2+1}\,dt - 2\int \frac{1}{t^2+1}\,dt = \frac{3}{2}\ln|t^2+1| - 2\arctan t + C$
$= \frac{3}{2}\ln((x+1)^2+1) - 2\arctan(x+1) + C = \frac{3}{2}\ln(x^2+2x+2) - 2\arctan(x+1) + C$

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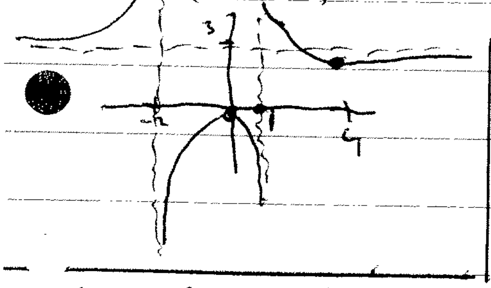
6) $f(x) = \frac{3x^2}{(x-1)(x+2)}$ $\quad D_f = \mathbb{R}\setminus\{-2, 1\}$ $\quad \lim_{x\to\pm\infty} f(x) = 3$, $\lim_{x\to -2^{\pm}} f(x) = \mp\infty$, $\lim_{x\to 1^{\pm}} f(x) = \pm\infty$

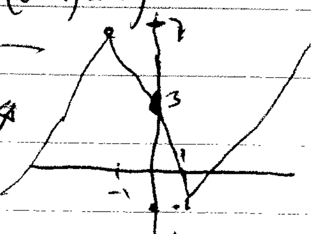
$f'(x) = \frac{6x(x^2+x-2) - 3x^2(2x+1)}{(x-1)^2(x+2)^2} = \frac{3x^2 - 12x}{(x-1)^2(x+2)^2} = \frac{3x(x-4)}{(x-1)^2(x+2)^2}$

x		-2		0		1		4	
f'	+	∤	+	0	−	∤	−	0	+
f	↗		↗		↘		↘		↗



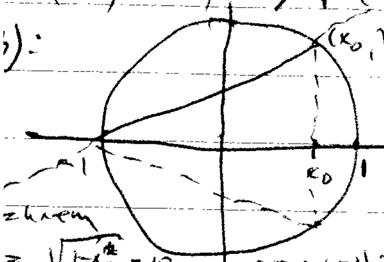
7) a): $f'(x) = 5x^4 - 5 = 5(x^4-1) = 5((x^2)^2-1) = 5(x^2-1)(x^2+1) =$
$= 5(x-1)(x+1)(x^2+1)$

x		-1		1	
f'	+	0	−	0	+
f	↗	7	↘	−1	↗



∴ f har invers på intervallet $[-1, 1]$ runt $x=0$ och detta är det största möjliga intervall runt $x=0$. Vidare är $f(0)=3$
så $(f^{-1})'(f(x)) = 1/f'(x)$ med $x=0$ ger $(f^{-1})'(3) = 1/f'(0) = 1/-5 = -\frac{1}{5}$

b):



$V(x_0) = \text{Rotationsvolym} = \int_{-1}^{x_0} \pi\left(\frac{\sqrt{1-x_0^2}}{x_0+1}(x+1)\right)^2 dx =$
$= \frac{\pi(1-x_0^2)}{(x_0+1)^2}\left[\frac{(x+1)^3}{3}\right]_{-1}^{x_0} = \frac{\pi}{3}\frac{1-x_0^2}{(x_0+1)^2}(x_0+1)^3 = -\frac{\pi}{3}(x_0^3 + x_0^2 - x_0 - 1)$

k-värdet $= \frac{\sqrt{1-x_0^2} - 0}{x_0 - (-1)} = \frac{\sqrt{1-x_0^2}}{x_0+1}$ $\quad 0 = y(-1) = k(-1) + m \Rightarrow m = k$ $\quad$ ∴ $y = kx + m = \frac{\sqrt{1-x_0^2}}{x_0+1}(x+1)$

$V'(x_0) = \frac{dV}{dx_0}(x_0) = -\frac{\pi}{3}(3x_0^2 + 2x_0 - 1) = -\frac{\pi}{3}(x_0+1)(3x_0-1)$

x	-1		$1/3$		1
V'	−	0 +	0	−	−
V	↘ 0	↗		↘ 0	↘

$\Rightarrow \max = V(1/3) = -\frac{\pi}{3}(1-\frac{1}{3})(\frac{1}{3}+1)^2 = \frac{32\pi}{81}$

8) Se kursboken sid 306 & 308