Naturvetarmatematik A1, MMGK11, Lösningsförslag 140612

1) a) $\int_0^{\pi} 6\cos(6x)\,dx = [\sin(6x)]_0^{\pi} = 0$ b) $\int \ln\sqrt{x}\,dx = \frac12\int \ln x\,dx = [PI] = \frac12\left(x\ln x - \int x\frac1x\,dx\right) = \frac12\left(x\ln x - \int dx\right) =$
$\frac12(x\ln x - x) + C = \frac12 x(\ln x - 1) + C$ c) $\int \sin(2x)\cos x\,dx = \left(\begin{array}{c}\text{produktformler}\\ \text{som fås från}\\ \text{additionsteoremen}\end{array}\right) = \int \frac12(\sin 3x + \sin x)\,dx$
$= -\frac16\cos 3x - \frac12\cos x + C$

2) IF: $e^{\int 1\,dx} = e^x \Rightarrow \frac{d}{dx}(e^x y) = xe^x \Rightarrow e^x y = \int \frac{d}{dx}(e^x y)\,dx = \int xe^x\,dx = [PI] = xe^x - \int 1\cdot e^x\,dx =$
$= xe^x - e^x + C = (x-1)e^x + C \Rightarrow y = x - 1 + Ce^{-x}$ och $2 = y(0) = 0 - 1 + Ce^0 \Rightarrow C = 3 \Rightarrow y = x - 1 + 3e^{-x}$

3) $y'' + 2y' + y = x$; $y = y_h + y_p$ Kar. ekv: $r^2 + 2r + 1 = 0 = (r+1)^2 \Rightarrow y_h = (Ax+B)e^{-x}$
Ansätt $y_p = x^m(C_1x + C_2) = (m=0) = C_1x + C_2 \Rightarrow y_p' = C_1$, $y_p'' = 0$ Insättning $\Rightarrow$
$0 + 2C_1 + C_1x + C_2 = x \Rightarrow C_1 = 1$, $C_2 = -2 \Rightarrow y_p = x - 2 \Rightarrow y = x - 2 + (Ax+B)e^{-x}$

4) $\lim_{x\to0}\frac{(e^x)^2 - \cos x}{\arctan x} = \lim_{x\to0}\frac{e^{2x} - \cos x}{\arctan x} = \lim_{x\to0}\frac{1+2x+O(x^2) - (1+O(x^2))}{x+O(x^3)} =$
$= \lim_{x\to0}\frac{x(2+O(x))}{x(1+O(x^2))} = \lim_{x\to0}\frac{2+O(x)}{1+O(x^2)} = \frac{2+0}{1+0} = 2$

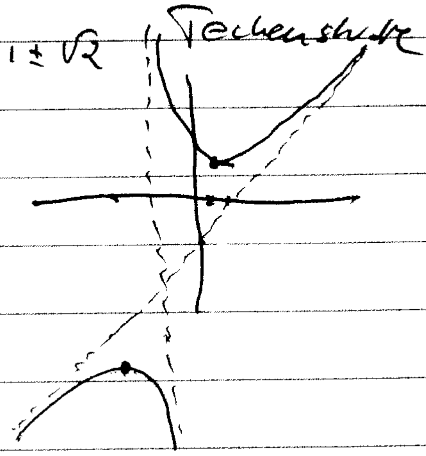
5) $\frac{x^3+2x^2+5x+1}{x^2+2x+2} = x + \frac{3x+1}{x^2+2x+2}$ och $\frac{3x+1}{x^2+2x+2} = \frac{3x+1}{(x+1)^2+1}$ så
$\int\frac{x^3+2x^2+5x+1}{x^2+2x+2}\,dx = \int x + \frac{3x+1}{x^2+2x+2}\,dx = \frac{x^2}{2} + \int\frac{3x+1}{(x+1)^2+1}\,dx = [t = x+1] = \frac{x^2}{2} + \int\frac{3(t-1)+1}{t^2+1}\,dt$
$= \frac{x^2}{2} + \int\frac{3t-2}{t^2+1}\,dx = \frac{x^2}{2} + \frac32\int\frac{2t}{t^2+1}\,dt - 2\int\frac{1}{t^2+1}\,dt = \frac{x^2}{2} + \frac32\ln(t^2+1) - 2\arctan t + C$
$= \frac{x^2}{2} + \frac32\ln(x^2+2x+2) - 2\arctan(x+1) + C$

6) $y = \frac{x^2+1}{x+1} = x - 1 + \frac{2}{x+1}$ så $y = x - 1$ är asymptot då $x \to \pm\infty$

$f'(x) = \frac{2x(x+1) - (x^2+1)\cdot 1}{(x+1)^2} = \frac{x^2+2x-1}{(x+1)^2} = 0 \Rightarrow x_{1,2} = -1 \pm \sqrt2$

$\lim_{x\to -1^{\pm}} f(x) = \pm\infty$ ∴

x		$-1-\sqrt2$		-1		$-1+\sqrt2$	
f'	$+$	0	$-$	$\nexists$	$-$	0	$+$
f	$\nearrow$	$-2\sqrt2-2$	$\searrow$		$\searrow$	$2\sqrt2-2$	$\nearrow$



7) $y = y_p + y_h$ Kar. ekv. $r^2 + 1 = 0 \Rightarrow r_{1,2} = \pm i \Rightarrow$
$y_h = D_1e^{-ix} + D_2e^{ix} = C_1\cos x + C_2\sin x$ Vidare är
$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} \Rightarrow$ lös $y_{p_1}'' + y_{p_1} = \frac12$ och $y_{p_2}'' + y_{p_2} = \frac12\cos x$
∴ Ansätt $y_{p_2} = x^m(A\cos x + B\sin x) = (m=1) = x(A\cos x + B\sin x) \Rightarrow y_{p_2}' = \dots$ och $y_{p_2}'' = \dots$ och
insättning i ekv. ~~och~~ identifiering av koeff. $\Rightarrow y_{p_2} = x\left(0\cos x + \frac14\sin x\right) =$
$= \frac{x}{4}\sin x$ Vidare är $y_{p_1} = \frac12$ ∴ $y = y_{p_1} + y_{p_2} + y_h = \frac12 + \frac{x}{4}\sin x + C_1\cos x + C_2\sin x$

8) Se kursboken.