Naturvetarmatematik A1, MMGK1, del 2 140829

1a) $\int_0^{\pi/10} \sin 5x\,dx = \left[\frac{-\cos 5x}{5}\right]_0^{\pi/10} = \dots = \frac{1}{5}$ b) $\int_0^{1} \frac{1}{x^2+2}dx = \frac{1}{2}\int_0^1 \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}dx = \left[\begin{matrix} t = x/\sqrt{2} & 0 \to 0 \\ dt = \frac{1}{\sqrt{2}}dx & 1 \to 1/\sqrt{2}\end{matrix}\right] = \frac{1}{2}\int_0^{1/\sqrt{2}} \frac{\sqrt{2}}{t^2+1}dt =$

$= \frac{\sqrt{2}}{2}\left[\arctan t\right]_0^{1/\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\arctan\frac{1}{\sqrt{2}} - \arctan 0\right) = \frac{1}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}}$ c) $\int x^3(1+x)dx = \int x^3 + x^4\,dx = \frac{x^4}{4} + \frac{x^5}{5} + C$

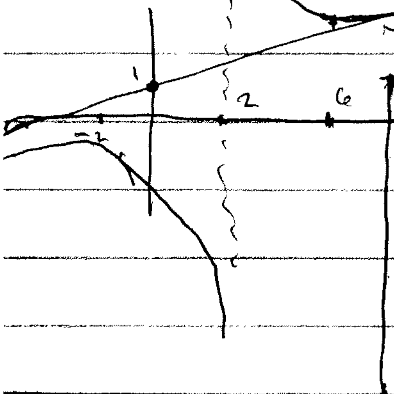
2a) IF: $e^{\int 1\,dx} = e^x \Rightarrow \frac{d}{dx}(e^x y) = xe^x \Rightarrow e^x y = \int \frac{d}{dx}(e^x y)dx = \int xe^x dx = [PI] = xe^x - \int 1 \cdot e^x dx =$
$= xe^x - e^x + C \Rightarrow y = x - 1 + Ce^{-x}$, $2 = y(0) = 0 - 1 + Ce^0 \Rightarrow C = 3 \Rightarrow$ sökt lösn. $y = x - 1 + 3e^{-x}$ b) IF: $e^{\int \frac{2}{x}dx} =$
$= e^{2\ln x} = e^{\ln x^2} = x^2 \Rightarrow \frac{d}{dx}(x^2 y) = x^2 \cdot x^2 = x^4 \Rightarrow x^2 y = \int \frac{d}{dx}(x^2 y)dx = \int x^4 dx = \frac{x^5}{5} + C \Rightarrow$
$y = \frac{1}{5}x^3 + Cx^{-2}$, $0 = y(-1) = -\frac{1}{5} + C \Rightarrow C = \frac{1}{5}$ $\therefore y(x) = \frac{1}{5}x^3 + \frac{1}{5}\frac{1}{x^2} \Rightarrow y(1) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

3) $y = y_p + y_h$ Kar. ekv. $r^2 + 2r + 1 = 0 = (r+1)^2 \Rightarrow r_{1,2} = -1 \Rightarrow y_h = (Ax+B)e^{-x}$ Ansätt
$y_p = x^m(ax+b) = (m=0) = ax + b \Rightarrow y_p' = a,\ y_p'' = 0 \Rightarrow 0 + 2a + ax + b = x \Rightarrow a = 1, b = -2 \Rightarrow y_p = x - 2 \therefore y = x - 2 + (Ax+B)e^{-x}$

4) $\lim_{x\to 0} \frac{x^2 + 2(\cos x - 1)}{x^2 - x\ln(1+x)} = \lim_{x\to 0} \frac{x^2 + 2\left(-\frac{x^2}{2} + \frac{x^4}{4!} + \mathcal{O}(x^6)\right)}{x^2 - x\left(x - \frac{x^2}{2} + \mathcal{O}(x^3)\right)} = \lim_{x\to 0} \frac{x^4\left(\frac{1}{12} + \mathcal{O}(x^2)\right)}{x^3\left(\frac{1}{2} + \mathcal{O}(x)\right)} = \lim_{x\to 0} x \lim_{x\to 0} \frac{\frac{1}{12} + \mathcal{O}(x^2)}{\frac{1}{2} + \mathcal{O}(x)} = 0 \cdot \frac{\frac{1}{12}+0}{\frac{1}{2}+0} = 0$

5) ● $D_f = \mathbb{R}\setminus\{2\}$ $f(x) = \frac{1}{2}\frac{x^2+12}{x-2} = \frac{1}{2}x + 1 + \frac{8}{x-2}$ $\therefore y = \frac{1}{2}x + 1$ sned asymptot, $x = 2$ lodrät asymptot. $f'(x) = \frac{1}{2}\left(\frac{2x(x-2) - (x^2+12)\cdot 1}{(x-2)^2}\right) = \frac{1}{2}\frac{x^2 - 4x - 12}{(x-2)^2} = \frac{1}{2}\frac{(x+2)(x-6)}{(x-2)^2}$

x		-2		2		6	
f'	$+$	0	$-$	$\nexists$	$-$	0	$+$
f	$\nearrow$		$\searrow$		$\searrow$		$\nearrow$



6) $y' = ry(K-y) \Rightarrow \frac{1}{y(K-y)}dy = r\,dt \Rightarrow$ [PBU] $\Rightarrow \frac{1}{K}\int \frac{1}{y} + \frac{1}{K-y}dy =$
$= \int r\,dt \Rightarrow \frac{1}{K}(\ln|y| - \ln|K-y|) = rt + C \Rightarrow \ln\left|\frac{y}{K-y}\right| = Krt + KC$
$\Rightarrow \left|\frac{y}{K-y}\right| = e^{Krt}e^{KC} = Ce^{Krt}$, $C > 0$
$\Rightarrow \frac{y}{K-y} = \pm Ce^{Krt} = Ce^{Krt}$, $C \neq 0 \Rightarrow$

● $y = CKe^{rKx}/Ce^{rKx} + 1$ och begynnelsevärde ger $K = 10^5$, $C = 1/9$, $r = \frac{1}{10^5}\ln\left(\frac{9}{4}\right)$

7) $y(t)$ = mängden jäst i kilo i schaktugnen vid tiden t
$y(0) = y_0$ kg

y' = förändring av $y(t)$ per tidsenhet $= ky - a$
där k är proportionalitetskonstanten $= 0{,}2$
$\therefore y' - ky = -a$ IF $= e^{\int -k\,dt} = e^{-kt}$ $\therefore \frac{d}{dt}(e^{-kt}y) =$
$= -ae^{-kt} \Rightarrow e^{-kt}y = \int \frac{d}{dt}(e^{-kt}y)dt = \int -ae^{-kt}dt =$
$= -a\left(\frac{e^{-kt}}{-k}\right) + C = \frac{a}{k}e^{-kt} + C$ $\therefore y = \frac{a}{k} + Ce^{kt}$
$\therefore \frac{a}{k} + Ce^0 = y(0) = y_0 \Rightarrow C = y_0 - \frac{a}{k} \Rightarrow \boxed{y = \frac{a}{k} + \left(y_0 - \frac{a}{k}\right)e^{kt}}$
Så om $y_0 - \frac{a}{k} = 0$ är mängden jäst konstant $\Leftrightarrow a = ky_0 = 0{,}2y_0$

8) Se kursboken