Analysseminariet

Ljudmila A. Bordag, Halmstad: Quasi periodic vortex structures in two-dimensional flows in an inviscid incompressible fluid

Abstract: (Work with Mikhail V. Babich, St. Petersburg) We investigate a two-dimensional case of a steady motion of an inviscid incompressible fluid. The most general form for an equation of the motion in this case is

$$\Delta u(x,y) = F(u(x,y)), \tag{1}$$

where u(x,y) is a stream function of local coordinates x, y, Δ is the Laplace operator and $F(\cdot)$ an arbitrary function. The function F is the measure of the flow vorticity. At present it seems that the only way to consider an equation of the type (1) with non trivial function F with analytical methods is to use the inverse scattering method. To the list of integrable equations of the form (1) belong the Sine - Laplace equation (SL), the Cos - Laplace equation, the Sinh -Laplace equation (SHL), the Liouville equation (an explicit integrable one), the Tzitzeica equation and the Cosh - Laplace equation. The first five of these equations allow a Lamb - substitution, which was used to describe the simplest types of a fluid motion. The characteristics of corresponding flows evince typical properties connected with the separation of variables in the equations. In terms of flow properties we can describe the flows in corner-like regions or in channels with straight lines boundaries.

The idea to look on the solutions of the Cosh - Laplace equation (CHL)

$$\Delta u(x,y) = \pm \cosh(u(x,y)), \tag{2}$$

came from this point of view. This equation doesn't admit the Lamb substitution, hence we can get a more interesting and complicated structure of singularities. It is one of the equations which

can direct us to a description of flows in regions with non strait line boundaries. The special technique of the finite-gap integration allows to get real solutions of the CHL equation using a compact Riemann surface with a proper symmetry.

We study the first non trivial case and take a Riemann surface of the genus $g = 3$. The hydrodynamical interpretations of the finite-gape solutions is non trivial and we try to reach a complete understanding of the described processes in fluid. To reach this goal we will take a Riemann surface with additional symmetry properties.

We present the four five parametric families of exact solutions. These solutions are given in term of Jacobian elliptic functions, which allows direct investigations of all physical properties. We were also able to found the explicit formulas for the singularities curves. From point of view of the algebraic geometry it is interesting to remark that it was possible to describe the complete structure of the real part of theta divisor.

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