MISSING TERMS IN HARDY-SOBOLEV INEQUALITIES AND ITS APPLICATION

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Abstract

We consider the Hardy-Sobolev inequalities of the following type:

$$\int_\Omega |\Delta u|^p dx \geq \left(\frac{n-2p}{p}\right)^p \left(\frac{np-n}{p}\right)^p \int_\Omega \frac{|u(x)|^p}{|x|^{2p}} dx$$

for any $u \in W_0^{2,p}(\Omega)$, where Ω a bounded domain in $\mathbb{R}^n$ with $0 \in \Omega$, $n \geq 3$, and $1 < p < \frac{n}{2}$. We improve this inequality by adding a term with a singular weight of the type $\left(\log \frac{1}{|x|}\right)^{-2}$. We show that this weight function is optimal in the sense that the inequality fails for any other weight functions more singular than this one. As an application, we use our improved inequality to determine exactly when the first eigenvalue of the weighted eigenvalue problem for the operator $L_\mu u = \Delta\left(|\Delta u|^{p-2}\Delta u\right) - \frac{\mu}{|x|^{2p}}|u|^{p-2}u$ will tend to 0 as μ increases to $\left(\frac{n-2p}{p}\right)^p \left(\frac{np-n}{p}\right)^p$.