Square Functions, Uniform Rectifiability and Regularity of Parabolic Free Boundary Problems

Kaj Nyström*
Department of Mathematics, Umeå University
S-90187 Umeå, Sweden

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Abstract

In this talk I will discuss the characterization of domains Ω which are δ_0-Reifenberg flat and Ahlfors in the parabolic sense via information on the caloric Poisson kernel. In particular I assume that $\log k(\hat{X}, \hat{t}, \cdot) \in VMO(\partial\Omega)$ and I want to understand if this information combined with a flatness assumption $\delta_0 \leq \delta_n$, for some δ_n small enough, implies that $\partial\Omega$ is a chord arc domain with vanishing constant in the parabolic sense. Here $k(\hat{X}, \hat{t}, \cdot)$ is the Poisson kernel, associated to the heat equation, with pole at $(\hat{X}, \hat{t})$. I will present some partial result on the problem stated above and in particular prove that the problem has an affirmative answer in case $\Omega \subset \mathbf{R}^2$. A key idea explored in the arguments is a blow-up argument and the classification of certain non-negative adjoint caloric functions u (Green functions with pole at infinity) on the limiting domain. A new component in the arguments is the proof of a certain square function estimate for the second order derivatives of the Green function u with pole at infinity and its implication on the regularity of the boundary in terms of uniform rectifiability. The results and tools presented represent steps towards a generalization, to the parabolic setting, of results of Alt-Caffarelli and Kenig-Toro concerning elliptic free boundary problems on (defined in the elliptic setting) Reifenberg flat, Ahlfors regular domains.

*email: kaj.nystrom@math.umu.se